



PLOVDIV UNIVERSITY „PAISII HILENDARSKI“

FACULTY OF MATHEMATICS AND INFORMATICS



**DEPARTMENT OF „EDUCATION IN
MATHEMATICS, INFORMATICS, AND INFORMATION TECHNOLOGIES “**

PENKA GEORGIEVA KARADZHOVA

**SYNERGETIC ASPECTS OF CONTINUITY IN MATHEMATICS
EDUCATION IN THE SECONDARY SCHOOL
(5TH – 7TH GRADE)**

**ABSTRACT
OF A DISSERTATION**

for the award of the educational and scientific degree „Doctor“ (PhD)

Field of Higher Education: 1. Pedagogical Sciences

Professional Field: 1.3. Pedagogy of Education in ...

Doctoral Program: Methodology of Mathematics Education

Scientific supervisor:

Assoc. Prof. Dobrinka Vasileva Milusheva-Boykina, PhD

Plovdiv, 2026

The dissertation was discussed and approved for public defense at an extended meeting of the Department of Education in Mathematics, Informatics and Information Technology at the Faculty of Mathematics and Informatics, Plovdiv University „Paisii Hilendarski“.

The dissertation entitled „**Synergetic aspects of continuity in mathematics education in the secondary school (5th – 7th grade)**“ comprises 256 pages, of which 178 pages constitute the main body, including an introduction and three chapters; 9 pages of references; and three appendices totaling 69 pages. The bibliography includes 144 references to books and articles and 10 internet sources. The cited articles and books are 80 in Cyrillic and 64 in Latin script.

The list of the author’s publications related to the dissertation includes five titles, which have been cited twice.

The public defense of the dissertation will take place on April 15, 2026, at a.m. in the Conference Hall of the New Building of Plovdiv University „Paisii Hilendarski“. The defense materials are available to interested parties at the Secretariat of the Faculty of Mathematics and Informatics, New Building, office 330, every working day from 8:30 a.m. to 5:00 p.m.

Scientific jury:

Chairman:

Prof. Ivaylo Peev Staribratov, PhD (Plovdiv University „Paisii Hilendarski“ – Plovdiv)

Members:

1. Prof. Darinka Nencheva Galabova, PhD (University of Veliko Tarnovo „St. Cyril and Methodius“ – Veliko Tarnovo)
2. Prof. Zdravko Vutov Lalchev, PhD (Sofia University „St. Kliment Ohridski“ – Sofia)
3. Assoc. Prof. Kamelia Bozhidarova Koleva, PhD (National Military University „Vasil Levski“ – Veliko Tarnovo)
4. Prof. Todorka Zhivkova Terzieva, PhD (Plovdiv University „Paisii Hilendarski“ – Plovdiv)

Author: Penka Georgieva Karadzhova

Title: Synergetic aspects of continuity in mathematics education in the secondary school (5th – 7th grade)

Circulation: 30 copies.

Plovdiv, 2026

Content

INTRODUCTION.....	4
1. RELEVANCE OF THE RESEARCH PROBLEM	4
2. MOTIVES FOR THE SELECTION OF THE RESEARCH TOPIC	4
3. OBJECT AND SUBJECT OF THE RESEARCH	4
4. GOALS AND OBJECTIVES OF THE DISSERTATION	4
5. OBJECTIVES OF THE DISSERTATION RESEARCH	5
6. MAIN RESEARCH HYPOTHESIS:	5
7. RESEARCH METHODS.....	5
8. RESEARCH METHODOLOGY	6
9. EXPECTED RESULTS AND CONTRIBUTIONS	6
10. STRUCTURE OF THE DISSERTATION	6
CHAPTER I. THEORETICAL FOUNDATIONS OF THE RESEARCH PROBLEM.....	7
1.1. CONTINUITY – ESSENCE AND TYPES OF CONTINUITY IN EDUCATION	7
1.2. CONTINUITY IN MATHEMATICS EDUCATION – SCIENTIFIC AND METHODOLOGICAL CONTEXT	8
1.3. SYNERGETICS – ESSENCE, PRINCIPLES, AND RELATION TO SELF-ORGANIZATION	10
1.4. ELEMENTS OF SYNERGETICS AND OPPORTUNITIES FOR THEIR APPLICATION IN SCHOOL MATHEMATICS EDUCATION	12
1.5. SYNERGETIC ASPECTS OF CONTINUITY IN MATHEMATICS EDUCATION (GRADES 5 TH – 7 TH)	13
CONCLUSIONS FROM CHAPTER ONE	14
CHAPTER II. TECHNOLOGICAL MODEL AND APPROACHES FOR IMPLEMENTATION IN PEDAGOGICAL PRACTICE ..	15
2.1. ANALYSIS OF THE CURRICULUM CONTENT AND METHODOLOGICAL PRACTICES IN MATHEMATICS EDUCATION (GRADES 5 TH –7 TH)	15
2.2. PSYCHOLOGICAL-PEDAGOGICAL CHARACTERISTICS OF ADOLESCENT DEVELOPMENT (AGES 11–14; GRADES 5 TH – 7 TH)	17
2.3. METHODOLOGICAL SPECIFICITIES OF EDUCATION IN LOWER SECONDARY EDUCATION	18
2.4. STRUCTURAL COMPONENTS OF THE TECHNOLOGICAL MODEL	20
2.5. MODULES IN THE PEDAGOGICAL EXPERIMENT	22
CONCLUSIONS FROM CHAPTER TWO.....	23
CHAPTER III. ORGANIZATION AND IMPLEMENTATION OF THE EMPIRICAL STUDY. RESULTS OF THE PEDAGOGICAL EXPERIMENT.....	24
3.1. ORGANIZATION OF THE EMPIRICAL STUDY	24
3.2. CRITERIA AND INDICATORS FOR DIAGNOSING THE RESULTS OF THE PEDAGOGICAL EXPERIMENT	24
3.3. IMPLEMENTATION ACTIVITIES OF THE PEDAGOGICAL EXPERIMENT	25
3.4. STUDY OF THE MAIN CHARACTERISTICS OF THE DIDACTIC MATERIALS	25
3.5. APPROBATION OF THE MODEL IN THE SCHOOL ENVIRONMENT	25
3.6. ANALYSIS OF RESULTS	25
3.7. CONCLUSIONS AND GENERALIZATIONS FROM THE EMPIRICAL STUDY	26
CONCLUSIONS FROM CHAPTER THREE	26
CONCLUSION.....	27
MAIN CONTRIBUTIONS OF THE DISSERTATION	28
RECOMMENDATIONS AND PROSPECTS FOR THE DEVELOPMENT OF THE TECHNOLOGICAL EDUCATIONAL MODEL.....	29
PUBLICATIONS RELATED TO THE DISSERTATION TOPIC.....	29
NOTED CITATIONS	30
ACKNOWLEDGEMENTS.....	30
BIBLIOGRAPHY	31

INTRODUCTION

1. Relevance of the Research Problem

The lower secondary stage (5th – 7th grade) is critical for the transformation of mathematical knowledge from an intuitive-practical to an abstract-theoretical level. It is precisely within this transition that both opportunities for conceptual advancement and risks of cognitive discontinuity emerge. In this context, there arises a need for a new perspective on the structuring and methodological implementation of continuity – one that takes into account not only the linear progression of knowledge but also the complex interactions among students, content, the learning environment, and teacher competence. This is where the potential of the synergetic approach is discovered, which conceptualizes learning as a self-organizing system susceptible to internal interrelations and nonlinear dynamics.

2. Motives for the Selection of the Research Topic

The selection of the lower secondary stage as the object of the present study is motivated by its pivotal role within the system of mathematics education and by the specificity of the cognitive, curricular, and methodological transitions that occur during this period. It is in Grades 5th – 7th that the shift from concrete-operational to formal-logical thinking takes place, as well as the transition from arithmetic to algebraic and functional thinking and from intuitive acquisition of knowledge to the conscious construction of abstract mathematical structures. During this stage, stable learning strategies, metacognitive skills, and attitudes toward mathematics are formed, which determine students' subsequent development; consequently, continuity in mathematics education at this level is both particularly vulnerable and fundamentally significant.

3. Object and Subject of the Research

The object of the pedagogical experiment comprises students currently engaged in mathematics instruction implemented in accordance with the curricula for the lower secondary stage (Grades 5th – 7th) approved by the Ministry of Education and Science.

The subject of the research includes **the synergetic aspects of continuity** in mathematics education, **the implementation of continuity** in mathematics education at the lower secondary stage, and **the methodological approaches** for its realization.

4. Goals and objectives of the dissertation

The main aim of the dissertation is to develop and experimentally verify *a model for the implementation of continuity* through a synergetic approach in mathematics education in Grades 5th – 7th, with the purpose of improving the quality of students' acquisition of mathematical knowledge and fostering their sustainable cognitive development.

5. Objectives of the Dissertation Research

In order to achieve the main aim, the following research objectives have been formulated:

- 1) To analyze the theoretical foundations of the concept of „continuity“ and its significance in mathematics education.
- 2) To examine the principles of the synergetic approach and the possibilities for its application in a pedagogical context.
- 3) To analyze the curriculum content and teaching methodology in Grades 5th –7th with regard to continuity.
- 4) To diagnose the actual state of continuity through surveys, observations, and testing.
- 5) To design a technological model for the implementation of **synergetic continuity**.
- 6) To conduct a pedagogical experiment to validate the model and to analyze the experimental results in order to formulate conclusions and practical recommendations.

6. Main Research Hypothesis: *The implementation of continuity in mathematics education at the lower secondary stage (Grades 5th –7th) through a synergetic approach fosters the development of **synergetic thinking** in students, enabling them to achieve a higher level of understanding, acquisition, and application of mathematical knowledge.*

Working Sub-Hypotheses:

- 1) The introduction of synergetic methodological principles in mathematics education facilitates the transfer of knowledge across grades and topics.
- 2) The systematic interaction among content, learning activities, and students' individual characteristics leads to the self-organization of students' knowledge.
- 3) Teacher reflection and adaptability are key factors in achieving effective continuity.

7. Research Methods

The study employs theoretical, empirical, and statistical methods, including:

- Review and analysis of pedagogical, psychological, and methodological literature, as well as curricular documentation relevant to the research problem, in order to establish the theoretical framework of the study.
- Observation of the instructional process, comparison, analysis and synthesis, modeling, theoretical generalization, group discussions, and interlocution.
- Application of accumulated personal professional experience in mathematics education, including in mathematics education based on a synergetic approach to ensure continuity at the lower secondary stage.
- Testing. Using an instrumentarium comprising a system of tasks designed to assess students' knowledge and skills.

- Pedagogical experiment.
- Statistical methods for data processing and analysis.

8. Research Methodology

The research is grounded in an **interdisciplinary approach**, integrating pedagogical, psychological, and synergetic theoretical foundations. The methodological framework includes:

- The principles of systematization, continuity, and purposeful organization of education.
- The concept of learning as a nonlinear, self-organizing system.
- An interaction-based model of learning encompassing activity, motivation, and reflection.

9. Expected Results and Contributions

The study is expected to lead to:

- ✓ The development and validation of *an effective model of synergetic continuity*.
- ✓ The identification of *existing deficiencies in continuity* within mathematics education in Grades 5th, 6th and 7th.
- ✓ The formulation of *methodological recommendations* for improving mathematics instruction.
- ✓ A contribution to *the methodology of mathematics education* through an interdisciplinary interpretation of a significant pedagogical problem.

10. Structure of the Dissertation

The dissertation comprises 256 pages, of which 178 constitute the main body, including: an Introduction; three chapters (theoretical; practice-oriented; empirical and analytical); a Conclusion; Main Contributions and Recommendations; 9 pages of references; and three Appendices totaling 69 pages.

The reference list includes 144 titles of books and articles and 10 online sources. Of the cited books and articles, 80 are in Cyrillic and 64 in Latin script. The list of the author's publications related to the dissertation consists of 5 titles, which have been cited twice.

The **Introduction** presents the relevance of the research problem, the object and subject of the study, its aims and objectives, the research hypothesis, the methods employed, and the structure of the dissertation.

Chapter I provides a systematic theoretical analysis of the concept of continuity in education and explores the possibilities for integrating the synergetic approach into mathematics education at the lower secondary level. It outlines contemporary trends in mathematics education, the factors influencing continuity, models of cognitive development, and the role of information and communication technologies, interdisciplinary connections, and contextualization.

Chapter II delineates the theoretical and applied framework of the technological model designed to ensure continuity in mathematics education through the implementation of a

synergetic approach in Grades 5th – 7th. The constructed model is presented, structuring the educational process into clearly defined modules: language and symbolism, problem-solving strategies, work with errors, digital technologies, and STEM context.

Chapter III presents the organization and methodology of the pedagogical experiment conducted in an authentic school environment with equivalent control and experimental groups. Criteria and indicators for diagnosing the results of the experiment are developed. Two complementary experimental designs – „control group–experimental group“ and „pre-test–post-test“ – are applied, and the results are analyzed.

The Appendices include the diagnostic tests administered, the corresponding results, and the didactic materials developed for the study.

CHAPTER I. THEORETICAL FOUNDATIONS OF THE RESEARCH PROBLEM

1.1. Continuity – Essence and Types of Continuity in Education

In contemporary educational discourse, continuity is regarded as a dynamic and multi-dimensional connection ensuring transition between different educational levels while guaranteeing coherence and consistency of the educational process. This renders the concept complex and interdisciplinary, requiring analysis within didactic, psychological and pedagogical, and organizational contexts.

1.1.1. Conceptual Foundations of the Concept of „Continuity“

The roots of the idea of continuity in education can be traced back to ancient philosophy and classical didactics. During the twentieth and twenty-first centuries, the concept of continuity has been enriched through ideas from cognitive psychology, constructivism, developmental theory, as well as interdisciplinary approaches such as systems analysis and synergetics. Continuity is no longer viewed merely as a linear chronology of knowledge, but as a dynamic, adaptive, and contextual process involving interaction among cognitive, emotional, social, and metacognitive factors.

1.1.2. Types of Continuity – Classifications and Pedagogical Dimensions

The most common typologies in pedagogical literature include:

- ❖ **According to educational stages:** vertical continuity; horizontal continuity.
- ❖ **According to the nature of connections:** content continuity; methodological continuity; psychological continuity; organizational continuity.
- ❖ **According to didactic objectives:** functional continuity; value-based continuity; reflective continuity.

1.1.3. Theoretical Models and Contemporary Interpretations

Contemporary approaches treat continuity as a dynamic process of pedagogical transformation. The implementation of the competence-based approach in education necessitates a new vision of continuity – as coherence in the development of key competences and transversal skills. It is not limited to the acquisition of knowledge, but involves the formation of the ability to apply, interpret, and critically reflect upon it.

1.1.4. Continuity and the Regulatory Framework in Bulgarian Education

The system of school education is based on consecution and gradual advancement as a fundamental principle. In the mathematics curricula for Grades 5th – 7th, continuity is implemented through thematic units, defined expected learning outcomes, and transferable knowledge. Nevertheless, numerous empirical studies indicate existing problems in the practical implementation of continuity, both in terms of content and teaching methodology.

1.1.5. Challenges to Continuity in STEM Disciplines

Mathematics and science education are particularly sensitive to issues of continuity due to the cumulative nature of their content, where each new concept builds upon prior knowledge and understanding. Disruptions or lack of clarity in a given topic often result in didactic and factual gaps that hinder the assimilation of subsequent material. Practice shows that lower secondary students frequently encounter difficulties in the transition from intuitive to formal mathematical concepts, which necessitates a more precise and coordinated methodological strategy.

1.2. Continuity in Mathematics Education – Scientific and Methodological Context

In article [14], continuity in education is examined as a system of interrelated „continuities“. According to V. Milushev and D. Frenkev, „Continuity between the subsystems of students' education is realized in several directions: continuity in educational objectives; continuity in educational content; continuity in organizational forms; continuity in methods; continuity in instructional tools“ [6].

Continuity is not limited to content connections between topics and grades; it also encompasses consistency in methods and approaches used; synchronization between students' cognitive capacities and didactic requirements; gradual transition from intuitive to formal representations; and development of analytical, abstract, and logical thinking skills.

At the core of effective continuity lies the understanding that mathematical knowledge possesses a strictly structured, cumulative, and interconnected nature. Any omitted link or

improperly mastered concept may lead to accumulated difficulties that impede higher levels of thinking and problem-oriented learning.

1.2.1. Historical and Pedagogical Roots of Continuity in Mathematics Education

A historical and pedagogical review of the development of ideas concerning continuity in mathematics education reveals its significant and evolving value within changing educational paradigms. From strictly deductive systems in Antiquity to contemporary synergetic and competence-based models, continuity remains fundamental to effective learning. Viewed historically, it represents not only a didactic principle but an evolving pedagogical idea aimed at ensuring quality education for every child within their developmental and social context.

1.2.2. Structure of Mathematical Knowledge and Opportunities for Advancement

The structure of mathematical knowledge is governed by principles of logical coherence, abstraction, and systematic organization and is hierarchically arranged, with each subsequent level presupposing mastery of the preceding one. From the perspective of **continuity**, this implies:

- **Vertical progression** – knowledge acquired in lower grades forms the basis for mastering content in higher grades.
- **Horizontal connections** – simultaneous topics (e.g., in algebra and geometry) mutually reinforce one another through shared principles and logical relations.
- **Spiral model of education** – revisiting previously studied content at a higher level with expansion and deepening of knowledge.

An important aspect of knowledge advancement is **didactic transformation** – adapting content to students’ age and cognitive characteristics, such as: introducing abstract concepts initially through concrete models and visualizations; when learning new topics, analogies with previously learned concepts are used; and encouraging interdisciplinary thinking.

1.2.3. Fundamental Principles and Preconditions for Continuity in Mathematics Education

Continuity in mathematics education requires deliberate planning, methodological justification, and sustainability in teaching, grounded in certain didactic principles (principle of sequence and logical coherence; principle of accessibility; principle of systematicity and integration; principle of activity and independence; principle of variability and adaptability; principle of reflection and feedback), organizational conditions, and psychological-pedagogical prerequisites (students’ cognitive development; learning motivation; educational environment and resources; teacher’s role; coherence of curricula).

Didactic and methodological mechanisms for ensuring continuity include:

- ❖ Diagnostic assessment at the beginning of the academic year.
- ❖ Revision and synthesis.
- ❖ Interdisciplinary connections.
- ❖ References and analogies to previously studied content.

Sustainable continuity is achievable when teaching is grounded in scientifically justified principles, supported by methodological clarity, motivational encouragement, and purposeful organization of content.

1.2.4. Risks of Disrupted Continuity in Mathematics Education

Disruption of continuity in mathematics education leads to accumulated conceptual gaps, fragmented knowledge, difficulties in understanding new content, and decreased motivation for learning. Methodological and content-related inconsistency between educational stages creates cognitive barriers and negative attitudes, hindering the development of mathematical thinking. Lack of coordination, systematic progression, and individualized educational trajectories compromises sustainable knowledge acquisition, thus necessitating strategic planning and the application of a synergetic approach to ensure coherence and integrity in instruction.

1.2.5. Approaches and Methodological Tools for Ensuring Continuity

Ensuring continuity in mathematics education requires a comprehensive didactic approach that integrates the logic of mathematical knowledge, students' developmental characteristics, and the teacher's pedagogical competence. It is realized through integrated, constructivist, and spiral approaches, differentiation and individualization, supported by appropriate diagnostic and methodological tools. Continuity represents a dynamic process of sequential knowledge and skill development, whose effectiveness depends on systematicity, coherence, and reflexivity in education.

1.3. Synergetics – Essence, Principles, and Relation to Self-Organization

The examination of synergetics in the present dissertation has not only theoretical significance but also an important methodological role. Through the analysis of synergetic principles and their pedagogical interpretation, the foundation is laid for constructing a comprehensive model of mathematics education aimed at ensuring continuity and internal coherence of the learning content, as well as dynamic adaptation to students' needs and capacities.

1.3.1. Origin and Development of Synergetics as a Scientific Paradigm

Synergetics emerged in the second half of the twentieth century as an interdisciplinary scientific paradigm studying processes of self-organization and the emergence of new properties in open, nonlinear, and dynamic systems. Formulated by H. Haken and further developed by numerous scholars, it establishes the principle that the whole is greater than the sum of its parts and finds application in various fields, including pedagogy. In an educational

context, synergetics serves as a methodological framework for analyzing processes of learning, interaction, and development, enabling explanation of the transition from unstable to stable cognitive structures and supporting the contemporary understanding of education as a dynamic, self-organizing system.

1.3.2. Key Concepts in Synergetics: Open Systems, Fluctuations, Attractors, Self-Organization

The four key concepts in synergetics – open systems, fluctuations, attractors, and self-organization – not only describe processes in nature but also possess strong heuristic value in educational contexts. The educational environment and the student are viewed as open systems in which small deviations (fluctuations) may initiate qualitative changes and lead to new stable cognitive patterns (attractors). Self-organization is a process through which *order, structure, and new behavior emerge spontaneously* from interactions among system elements without central control or external imposition. In the learning process, self-organization manifests when *students begin to regulate their behavior, learning, and collaboration independently*, constructing sustainable learning strategies.

1.3.3. Principles of Synergetics and Their Interpretation in a Pedagogical Context

The principles of synergetics (homeostasis, hierarchy, nonlinearity, openness, instability, dynamic hierarchy, and observability) reveal mechanisms of transition between stability and development and vice versa, where imbalance and chaos are not perceived as destructive but as potential for new order and new quality. The principles of homeostasis and hierarchy describe stable states of the educational system, whereas nonlinearity, instability, and openness reveal mechanisms of qualitative transitions and development. Instability and fluctuations within the learning process are interpreted as resources for change and formation of new stable cognitive structures. The principle of observability substantiates the active role of the subject and positions the teacher as a facilitator of conditions for autonomy and self-organization. This approach provides a methodological foundation for conceptualizing and ensuring continuity in mathematics education.

1.3.4. Application of Synergetics in Pedagogy and Education – Overview of Theoretical Approaches

Various educational models [3], [7], [10], [16], [18], inspired by synergetics, **are examined**. These models challenge traditional linear and standardized conceptions of education by allowing spontaneity, individual developmental trajectories, and creative restructuring of outdated cognitive frameworks.

1.4. Elements of Synergetics and Opportunities for Their Application in School Mathematics Education

1.4.1. Possibilities for Transferring Synergetic Ideas into Mathematics Didactics

The synergetic approach enables a new interpretation of the goals of mathematics education – not merely acquisition of knowledge and skills, but development of students' capacity for self-organization in thinking, creative response to non-standard situations, and construction of their own cognitive structures in response to learning challenges. In accordance with the synergetic model, small pedagogical interventions – a well-chosen task, a guiding question, a provocative problem – may function as decisive fluctuations leading to qualitative leaps in understanding.

1.4.2. Mathematics Education as a Nonlinear and Open System

The process of mathematical thinking involves phases of exploration, trial and error, which activate mechanisms of self-organization and resemble the dynamics of complex systems. Synergetics interprets cognitive difficulties and states of instability as bifurcation points, where alternative developmental paths become possible. These boundary states between order and chaos prove essential for deep, sustainable, and meaningful acquisition of mathematical knowledge, as minor pedagogical influences may result in qualitative transformations in understanding.

1.4.3. Self-Organization in the Mathematics Learning Process

Students may attain new cognitive structures, profound understanding, and intellectual growth not solely as a result of one-directional teaching, but through active participation in the learning process and interaction with content, peers, and teacher. During this process, mechanisms of self-organization are activated, enabling students to regulate their thinking strategies and adapt to cognitive challenges. Self-organization manifests not only at the individual but also at the group level, when the classroom functions as an open system providing opportunities for choice, dialogue, and exchange of ideas, leading to sustainable patterns of collaborative work and collective construction of mathematical knowledge.

1.4.4. Synergetic Models of Teacher–Student Interaction

Dialogue, positive feedback, and productive instability function as mechanisms of self-organization and development, consistent with the ideas of [2], [25] and constructivist didactics. The teacher creates conditions for fluctuations and cognitive breakthroughs, while the student acts as an active subject with their own cognitive dynamics. The classroom is interpreted as an open, self-organizing system in which interaction and collective intelligence give rise to stable cognitive structures. The synergetic approach supports the realization of

continuity in education at the level of thinking, understanding, and transfer of cognitive models and requires the formation of a synergistically thinking personality in the teacher

1.4.5. Advantages and Challenges in Implementing Synergetic Elements in Mathematics Education

Despite associated pedagogical, methodological, and institutional difficulties, the synergetic approach in mathematics education entails substantial reconsideration of the teacher's role, the learning environment, and assessment practices. Its implementation reveals significant potential for fostering metacognitive skills, self-regulation, and a sustainable learning culture in which errors function as resources for self-organization and cognitive development. The application of synergetics in mathematics education represents not merely a pedagogical choice but *a philosophy of interaction and development*, wherein learning is viewed as a living, open, and emergent process, in harmony with the natural and social dynamics of reality [4], [17].

Within the context of the present dissertation, *synergetic thinking* is defined as a form of mathematical thinking characterized by nonlinear organization, self-organization, and functional interconnectedness of knowledge, enabling flexible application of acquired mathematical concepts and strategies in new learning situations. And *synergetic continuity* is understood as a didactically organized process of continuous and nonlinear advancement of knowledge, in which the interaction among learning content, students' prior experience, and methodological decisions leads to self-organization and sustainable development of learners' mathematical representations.

1.5. Synergetic Aspects of Continuity in Mathematics Education (Grades 5th – 7th)

1.5.1. Interrelation between the Synergetic Approach and the Idea of Continuity

Continuity is interpreted not as a linear sequence of topics but as a dynamic, sensitive, and adaptive process in which synergetic mechanisms – self-organization, instability, and emergence – ensure transition to higher levels of mathematical thinking. Within this framework, the teacher acts as a mediator of development, creating conditions for the student's internal cognitive reorganization and supporting the emergence of sustainable educational structures.

1.5.2. Continuity as a Process of Self-Organization in Mathematics Education

Within the synergetic paradigm applied to education, continuity is viewed as the result of multiple nonlinear interactions among prior and new knowledge, learning strategies, motivation, and students' cognitive capacities. Fluctuations can be identified in transitional moments when students encounter difficulties, misunderstandings, or resistance in the

learning process. These represent signals for the search for new stable states (attractors) leading to cognitive stabilization.

1.5.3. A Synergetic Reading of the Curriculum for Grades 5–7

A synergetic reading of curricula does not imply mechanical rewriting but rethinking them so as to emphasize recurrence, deepening, and internal interconnectedness of topics. Successful implementation of the synergetic perspective depends not only on content structure but also on flexibility of educational approaches, capacity for dialogue, and open learning that respects the internal dynamics of learning.

1.5.4. The Teacher's Role as Mediator of Synergetic Continuity

Within the synergetic paradigm, the teacher is not merely a transmitter of knowledge but an architect of an educational ecosystem in which continuity develops organically through the internal logic of content and students' needs. By managing instability, constructing meaningful connections, and supporting self-organization, the teacher contributes to sustainable and developmental mathematics education.

1.5.5. Examples of Applying Synergetic Strategies in Lower Secondary Education

Methodology should represent not merely a sequence of lessons but a model of an evolving learning environment in which the student is an active subject:

- Project-based approach with nonlinear thematic structure.
- Deliberate generation of „fluctuations“ and spontaneous shifts during lessons.
- Strategies for collective self-organization.
- Integration of intuitive and analytical thinking.
- Transitional lessons between grades with a cumulative structure.

CONCLUSIONS FROM CHAPTER ONE

- 1) ***Continuity*** in mathematics education should be regarded as a systemic and procedural phenomenon in which knowledge is transmitted, expanded, and restructured through interaction among student, teacher, content, and educational environment. This understanding transcends the traditional linear model and presupposes dynamic adaptation to students' cognitive development.
- 2) ***The synergetic paradigm*** provides an adequate conceptual apparatus for understanding education as a nonlinear, open, and self-organizing system. In this context, the educational process is characterized by states of instability, fluctuations, and emergence of new stable structures, consistent with contemporary educational requirements.
- 3) ***Continuity may be interpreted through synergetic principles*** as a naturally emerging connection between prior and new knowledge, conditioned not only by logical sequence

but also by internal cognitive necessity, network interconnectedness, and contextual relevance.

- 4) ***The learning process*** in lower secondary education unfolds through phases of transition and stabilization, with transitional moments playing a key role in constructing sustainable knowledge. Fluctuations generated by new topics or approaches may catalyze cognitive transformation and activate mechanisms of self-organization.
- 5) Within the synergetic model, ***the teacher*** performs not merely a controlling function but acts as a mediator who maintains dynamic equilibrium and creates conditions for the emergence of learning structures through dialogue, collaboration, and constructive uncertainty. This implies a change in pedagogical roles and the need for new didactic skills.
- 6) ***The learning content*** in mathematics for grades 5th – 7th allows a synergetic interpretation in which thematic progression is viewed through the prism of „attractors“ – central ideas or conceptual nodes around which knowledge is structured. Identifying such cores facilitates purposeful construction of continuity and conceptual coherence.
- 7) ***The implementation of synergetic strategies in practice*** (cooperative learning, open-ended problem solving, interdisciplinary connections, project-based learning, etc.) demonstrates increased effectiveness in depth of understanding, intrinsic motivation, and sustainability of knowledge. These strategies activate both individual and collective intelligence in the classroom.
- 8) In summary, ***the synergetic approach*** provides an innovative and scientifically grounded framework for designing the learning environment in accordance with the natural dynamics of cognitive processes and for building continuity through internal development rather than external imposition or formal transition.

CHAPTER II. TECHNOLOGICAL MODEL AND APPROACHES FOR IMPLEMENTATION IN PEDAGOGICAL PRACTICE

2.1. Analysis of the Curriculum Content and Methodological Practices in Mathematics Education (Grades 5th–7th)

2.1.1. Structure and Logic of the Mathematics Curriculum in Lower Secondary Education

The curriculum content in Grades 5th – 7th is well organized for a synergetic reading. The thematic units and the logic of cumulative progression create conditions for constructing a coherent, self-organizing, and adaptive mathematical system in students' minds. The next step is methodological – implementing these structural opportunities through appropriate pedagogical and technological solutions.

2.1.2. Methodological Approaches and Didactic Practices Used in Lower Secondary Education

Methodological practices at the lower secondary level evolve from predominantly algorithmic instruction toward more flexible, student-centered strategies. Their combination and strategic use within an appropriate context enable synergy between teaching, content, and student. This constitutes the foundation for sustainable continuity in mathematics education – a continuous which is logical and interconnected process of acquiring knowledge, skills, and competencies, in which each subsequent learning step builds meaningfully, comprehensibly, and functionally upon the previous one.

2.1.3. Analysis of Contemporary Mathematics Textbooks and Teaching Aids for Grades 5th – 7th

The analysis indicates that although contemporary textbooks approximate principles of modern education – structured presentation, visual support, and systematic organization – they do not yet fully meet the requirements of the synergetic approach, which presupposes greater nonlinearity, self-organization, reflection, and active student participation. Supporting synergetic continuity requires additional development of instructional materials that foster intuitive, inquiry-based, and interdisciplinary learning combined with opportunities for independent structuring of knowledge.

2.1.4. Opportunities for Applying the Synergetic Approach within Existing Practices

The synergetic approach does not necessitate dismantling existing methodologies but rather further developing them through integration of principles such as self-organization, dialogicity, adaptability, and systemic coherence. The potential for such transformation is already embedded in practice, provided it is reconsidered through a new methodological lens. Applying this approach may enhance motivation, autonomy, and meaningful continuity in mathematics education.

2.1.5. The Need for Improvement of Methodological Practices in the Context of Synergetic Didactics

Contemporary mathematics didactics possesses potential for synergetic reorientation; however, this requires a conscious shift in understanding the teacher's role, the functions of tasks, and classroom dynamics. It involves reflection on:

- 1) limitations of traditional methodological approaches;
- 2) synergetic criteria for effective methodological practice;
- 3) integration of feedback as a „regulatory mechanism“;
- 4) the need for interdisciplinary and contextualized tasks;
- 5) educational technologies as mediators of synergy.

2.2. Psychological-Pedagogical Characteristics of Adolescent Development (Ages 11–14; Grades 5th – 7th)

2.2.1. Characteristics of Cognitive Development in Pre- and Early Adolescence

Psychological studies [13], [20] analyzing the transition to formal operations through tasks requiring hypothetical-deductive reasoning demonstrate that students in this age range begin moving from the stage of concrete operations to formal logical operations. According to R. Siegler, cognitive development may be illustrated through the metaphor of „overlapping waves“, whereby „the use of different strategies increases and decreases over time“ [24]. This perspective resonates with synergetic theory, according to which cognitive development is a nonlinear process characterized by fluctuations, leaps, and new equilibria [12].

2.2.2. Emotional and Social Development of Lower Secondary Students

Neuropsychological studies [1] indicate significant restructuring of the prefrontal cortex and limbic system during this period, leading to heightened emotional sensitivity, difficulties in self-regulation, but also increased interest in cooperation and social learning. Teachers who demonstrate emotional support, attention to individual needs, and promotion of an inclusive culture create conditions for better mastery of academic content and for building a stable positive attitude toward the subject, particularly mathematics.

2.2.3. Motivation and Learning Strategies during the Transitional Period

Students who believe intelligence develops through effort cope more effectively with challenges, build resilience, and regulate their own learning. C. Dweck emphasizes the importance of providing a supportive environment that encourages effort rather than solely outcomes [5]. According to Zimmerman [26], self-regulated learning comprises three phases: planning, monitoring, and self-reflection – each actively developing during early adolescence. Encouraging metacognitive skills by the teacher is essential for their development.

2.2.4. Specific Features of Mathematics Education in Accordance with Age Characteristics

According to Piaget & Inhelder [21], the stage of formal operations begins around age 11, although for many children this transition is gradual and incomplete. Consequently, mathematics education should employ didactic strategies integrating concrete and abstract elements – through visualization, modeling, and connections to real-life contexts. The teacher must mediate the transition between concrete and abstract, ensuring smooth didactic continuity. Studies [22], [23] show that autonomy and relatedness are key motivators for students in early adolescents.

2.2.5. A Synergetic Approach to Personality Development and Learning

The synergetic approach regards the student's personality not as a static structure but as an open, self-organizing system in which learning results from complex nonlinear interactions among cognitive, emotional, social, and cultural factors. The synergetic perspective presupposes a didactics in which instruction adapts to the current state of the system „student“, with the teacher acting as mediator rather than controlling agent. As a new educational paradigm, synergetics in education encourages active participation, integration of knowledge, and development of complex competencies.

2.3. Methodological Specificities of Education in Lower Secondary Education

2.3.1. Didactic Principles and Objectives in Mathematics Education (Grades 5th – 7th)

The objectives and principles of mathematics education should be considered within a didactic ecosystem in which synergy between content, methods, and age specificity ensures meaningful and sustainable continuity. The dissertation presents our perspective on the possibilities for synergetic application of *didactic principles (scientific rigor, accessibility, systematicity and consistency, activity and independence, consciousness and durability, visualization, individualization and differentiation, connection with life and practice, interdisciplinarity)* in Grades 5th – 7th.

2.3.2. Selection and Application of Teaching Methods

Methodology in lower secondary education (5th – 7th) is crucial for achieving effective, engaging, and sustainable mathematical learning. Teachers must select a combination of traditional and innovative methods aligned with the didactic objectives, content, and students' developmental characteristics. The dissertation proposes appropriate methods (explanatory-illustrative, reproductive, problem-based, inquiry-based, game-based, project-based learning, discussion and debate approaches) and their application in Grades 5th – 7th.

2.3.3. The Role of Learning Tasks and Examples in Developing Mathematical Thinking

Learning tasks constitute a fundamental didactic tool in mathematics education. We distinguish the following types of tasks and their role in the thought process:

- Routine algorithmic tasks – develop automation and accuracy.
- Open-ended tasks – stimulate divergent thinking.
- Problem tasks – activate strategic and critical thinking.
- Modeling tasks – support transfer to real contexts.
- Interactive tasks – enhance participation through collaboration and communication.

Learning tasks in the lower secondary school stage should be treated not merely as exercises but as active mechanisms for cognitive and metacognitive development. According to Milusheva-Boykina and Milushev, constructing mathematical tasks represents a specific form of cognitive activity leading to deeper conceptual understanding and logical relationships in mathematics. They propose a reflexive-synergetic approach integrating problem solving and problem posing with cognitive activity and development of thinking at school [19].

2.3.4. Working with Errors, Difficulties, and Feedback

In contemporary mathematics education (Grades 5th – 7th), errors are valuable indicators of how students structure knowledge rather than mere absence of knowledge. Errors should be analyzed causally, identifying conceptual misunderstandings, strategic misjudgments, or transfer deficiencies. The synergetic approach interprets student errors as fluctuations within an open system, where knowledge stability is achieved through feedback and adaptation. From this point of view, feedback functions as a regulator supporting stabilization at a new level of cognitive organization.

Regarding the types of formative assessment and the elements included in traditional assessment, we agree with the opinion of Grozdev & Kirilova, who emphasize that „these two types of assessment are not mutually exclusive and in no way oppose one another“ [11].

2.3.5. The Role of ICT and Digital Resources in mathematics education

Information and communication technologies (ICT) play an increasingly significant role in mathematics education, particularly in lower secondary grades [8], [9]. They enable opportunities for interactive presentation of content, visualization of abstract concepts, personalization of learning process, and enhancement of student engagement. Mathematics curricula Grades 5th – 7th encourage ICT application in modeling, data representation, and real-world problem solving. Digital tools support development of key competencies, including mathematical literacy, digital competence, and lifelong learning.

2.3.6. Importance of Interdisciplinary Connections and Context

Interdisciplinary connections align with constructivist ideas represented in constructivism – knowledge is actively constructed through linking new information with prior knowledge – and with educational synergetics, where integration across academic disciplines enhances cognitive coherence and activates multiple cognitive channels.

2.4. Structural Components of the Technological Model

The technological model developed in this dissertation and presented in publication [15] is based on the idea for *dynamic interaction among content, methods, forms, and educational tools*, forming a *highly adaptive learning system* responsive to changes in the educational environment and students' individual needs.

2.4.1. Conceptual Foundation of the Model

The conceptual foundation of the technological model for synergistic continuity in mathematics education is built on several interconnected theoretical and methodological frameworks that provide scientific validity, pedagogical applicability, and the possibility of empirical validation and includes:

- 1) *Synergetic approach to education*
- 2) *Continuity as a pedagogical and systemic principle*
- 3) *Theories of thinking and learning development*
- 4) *Didactic principles and methodological continuity*
- 5) *Focus on competency formation.*

2.4.2. Objectives and Didactic Functions of the Model

The primary objective of the model is to create conditions for self-organization and sustainable formation of mathematical knowledge and competencies through by implementing a systemic, nonlinear, integrative approach, *synergetic approach*.

Didactic functions of the model include: motivational, cognitive, regulatory, differentiating, reflective, integrative, and prognostic.

2.4.3. Components of the Technological Model

The technological model, built on the synergistic paradigm, includes interconnected components that ensure its systematicity, flexibility and effectiveness in the conditions of mathematics education in Grades 5th – 7th. The main components of the model (*Fig. 1.*) are:

- Target,
- Content,
- Procedural,
- Diagnostic,
- Reflective,
- Organizational,
- Motivational.

Fig. 1. Schematic representation of connections



Each of these components interacts with the others, forming a dynamic system capable of adapting to the specific learning conditions, the characteristics of the students, and the specificity of the mathematical content. At the same time, they maintain and deploy the synergistic qualities of the model – interaction, sensitivity to initial conditions, sudden changes, and stabilization in new equilibria of learning.

2.4.4. Principles of Construction and Functioning of the Model

The main principles (of continuity, of self-organization, of non-linearity, of adaptability and flexibility, of active and collaborative learning, of meaningful feedback, of systematicity and integrativity) form the theoretical framework that determines the way of designing, implementing and evaluating the model. They ensure its synergistic nature and its potential to support simultaneously the cognitive, social and personal development of students.

2.4.5. Relationship between the Model and the Curriculum in Mathematics (Grades 5th – 7th) – Parameters for Pedagogical Implementation

The relationship between the model and the content is realized at several levels: conceptual, structural and procedural. Thus, the proposed technological model is not imposed externally on the content, but arises from its logic, supports the learning objectives and contributes to the development of deep, connected and transferable mathematical knowledge in students from 5th to 7th grade. The technological model aims to cause synergistic effects, in which the interaction between the individual methodological components leads to a qualitatively new educational state. Improved continuity between classes, increased student engagement, enhanced critical and abstract thinking, as well as sustainable internal motivation for learning mathematics are expected. The model activates self-organizational processes in learning through adaptive support and dynamic knowledge building.

In the technological model, the thematic core „Numbers. Algebra“ is organized as a **spirally developing system of interconnected concepts**, rather than as a linear sequence of individual topics. The main algebraic ideas – numerical dependence, symbolization, generalization and modeling – are introduced gradually and incrementally, in accordance with the age and cognitive characteristics of the students. **The Appendices** show possibilities for practical implementation of the technological model through the thematic core „Numbers. Algebra“, tracking the development of the concept of „number“ in the lower secondary school stage (Grades 5th – 7th). Sample tasks with methodological comments to them are presented, demonstrating the synergistic approach, continuity and spiral development of knowledge.

2.5. Modules in the Pedagogical Experiment

2.5.1. Structure and Objectives of the Modular Approach

The modular approach in the pedagogical experiment was chosen as the basis of the didactic organization due to its flexibility, ability to adapt to the needs of students and compliance with the synergistic learning model. The module as a structural unit offers completeness of educational content, combining cognitive, methodological and communicative aspects, which facilitates the targeted impact on the formation of mathematical competencies and guarantees continuity between individual school grades. The modular approach in this context is not only a technical organization of content, but a didactic framework, combining structurality, flexibility and transdisciplinarity, in the service of the sustainable acquisition of mathematical knowledge and skills in the lower secondary school stage.

2.5.2. Module 1: Continuity in the language and symbolism of mathematics

Module 1 is aimed at the smooth and conscious introduction, consolidation and upgrading of mathematical language and symbolism, so that students build confidence in its use, both orally and in writing. The goal is not only to correctly reproduce terms, symbols and signs, but also to understand their function as carriers of mathematical knowledge.

2.5.3. Module 2: Strategies for solving problems and developing mathematical thinking

Developing mathematical thinking in the lower secondary stage requires conscious work on building problem-solving strategies that are not only learned but also transferable to new contexts. *Module 2* focuses on ensuring continuity in approaches to problems, building on basic skills from the initial stage and preparing students for the abstract and analytical thinking characteristic of the upper grades.

2.5.4. Module 3: Learning errors, difficulties and adaptive intervention

The educational process in the lower secondary stage is saturated with cognitive challenges related to the expansion of the learning content and the transition to more abstract mathematical structures. In *Module 3*, errors and difficulties are viewed as opportunities for learning, self-awareness and pedagogical support.

2.5.5. Module 4: ICT and digital environments in synergy with mathematical content

In *Module 4*, the integration of ICT and digital environments are considered as elements in an open, dynamic educational system that interacts with the cognitive, emotional and social aspects of learning.

2.5.6. Module 5: Cross-curricular connections and contextualization of knowledge

Within the framework of the synergistic approach, *Module 5* aims to establish an active and conscious interconnection between mathematics and other academic disciplines, as well as with practical situations from everyday life, science and technology.

2.5.7. Module 6: Performance indicators and reflective evaluation of the modules

The effectiveness of the developed and tested modules in the pedagogical experiment is assessed through a system of qualitative and quantitative indicators that reflect both the achieved learning outcomes and the degree of engagement, understanding and development of key competencies in students. The approach in *Module 6* is reflexive, i.e. it includes not only external diagnostics, but also internal assessment through self-assessment, metacognitive awareness and feedback.

CONCLUSIONS FROM CHAPTER TWO

- 1) Mathematics education at the lower secondary level requires didactic solutions that combine continuity, adaptability and innovation. The specificities of the age group (11-14 years) require careful construction of methods and tools that simultaneously build on previous knowledge and prepare students for higher cognitive levels.
- 2) The psychological and pedagogical characteristics of students in grades 5th – 7th (cognitive instability, transition from concrete to abstract thinking, search for meaning and motivation) must be integrated into teaching as a leading factor. This requires dynamic learning strategies, active participation, and a variety of activities and approaches.
- 3) The methodological features of the training at this stage include the application of a wide range of didactic methods – from classical to digital – depending on the goals set and the characteristics of the students. Learning tasks, errors and ICT tools are used not only as means, but also as resources for building understanding, thinking and self-reflection.
- 4) The presented technological model is built on a solid conceptual foundation, including synergistic principles, didactic goals, a clear structure and a connection to the educational content. The components of the model are consistent both with each other and with the realities of school practice.
- 5) The modular approach provides an effective framework for achieving synergistic effects in mathematics education. Each module has its own specific role and contribution to the overall system and the overall interaction between them leads to increased motivation, deepened understanding, better adaptation and sustainable knowledge.
- 6) The synergy between methods, content, technologies and interpersonal dynamics (teacher-student, student-student) is a key factor for the success of the model. It is not achieved through mechanical matching, but by creating conditions for self-organization, feedback and a developing pedagogical environment.

CHAPTER III. ORGANIZATION AND IMPLEMENTATION OF THE EMPIRICAL STUDY. RESULTS OF THE PEDAGOGICAL EXPERIMENT

3.1. Organization of the Empirical Study

The approbation of the model in a real school environment aims to determine the extent to which its implementation leads to the development of sustainable cognitive motivation among students; enhancement of mathematical thinking; and improvement of academic achievement. The experiment also seeks to verify the validity of the working hypothesis formulated at the beginning of this dissertation research.

The pedagogical experiment was conducted over three consecutive academic years with six classes from the same student cohort (a total of 136 students) at “St. Konstantin-Kiril Filosof” Secondary School. The students were divided into two groups – experimental and control. The objective was to monitor the development of results resulting from the application of the proposed methodology throughout the entire period of mathematics education in lower secondary education.

To achieve this objective, the following tasks were formulated and implemented:

- 1) To organize a pedagogical experiment in a real school environment covering three consecutive academic years (Grades 5th – 7th).
- 2) To form experimental and control groups with an equal number of students and equivalent initial levels of mathematical achievement.
- 3) To implement the constructed technological model in the experimental group, while traditional methods were applied in the control group.
- 4) To conduct entry, intermediate, and final tests to assess student achievement in both groups.
- 5) To analyze and compare test results in order to evaluate the effectiveness of the model.

Data for analysis were derived from specially developed test instruments administered at key stages of education – at the beginning of Grade 5th (entry level), at the end of Grade 5th (formative experiment 1.), at the end of Grade 6th (formative experiment 2.), and at the end of Grade 7th (exit level). This approach allows both monitoring of progress and identification of sustainable trends in the acquisition of mathematical content.

3.2. Criteria and Indicators for Diagnosing the Results of the Pedagogical Experiment

A system of criteria and indicators was formulated in alignment with the objectives and hypothesis of the dissertation. These reflect both the level of mastery of mathematical knowledge and the degree of understanding and ability to apply knowledge, enabling a comprehensive evaluation of the effectiveness of the proposed synergetic model. Indicators

corresponding to each criterion are associated with quantitative measures and allow comparison between control and experimental groups.

3.3. Implementation Activities of the Pedagogical Experiment

The experiment was carried out under real school conditions in accordance with the principles and methodology outlined in the research framework. For each of the studied periods (Grades 5th, 6th, and 7th), the traditional three-stage structure of a pedagogical experiment was followed: preliminary (diagnostic) stage, formative stage, and final stage. To achieve higher objectivity and reliability of the results, two types of pedagogical experiments were applied within the framework of the study – a „*control group – experimental group*“ experiment and an „*entry level – exit level experiment*“.

3.4. Study of the Main Characteristics of the Didactic Materials

The didactic materials used in the experimental education were designed to ensure full implementation of the technological model developed in Chapter II and to support achievement of the goals set in the experiment. Their characteristics were examined with regard to didactic adequacy, motivational potential, logical and linguistic clarity, possibilities for differentiation, and practical applicability.

3.5. Approbation of the Model in the School Environment

The model developed in Chapter II was implemented in a real educational setting over three consecutive academic years – from the beginning of Grade 5th to the end of Grade 7th. The primary objective of the approbation was to assess the model’s applicability within mainstream schooling and to evaluate the outcomes achieved through its systematic implementation.

3.6. Analysis of Results

Comparing results from control and experimental groups is a fundamental approach in evaluating the effectiveness of a new educational methodology or pedagogical strategy. When two groups of students undergo different educational conditions, comparison of their results allows conclusions to be drawn regarding the success of the applied methodology.

3.6.1. Analysis of the Preliminary (Diagnostic) Test Results

Comparison between the control and experimental groups revealed no statistically significant differences according to either parametric or nonparametric criteria, and Cohen’s *d* (EG – CG effect size) was practically zero. This supports the conclusion that both groups were equivalent with respect to the examined knowledge and skills prior to the introduction of the technological model of education. Consequently, any differences identified in the exit

diagnostics at the end of Grade 7th may be interpreted with a high degree of justification as effects of the applied experimental methodology.

3.6.2. Analysis of the Final Experiment Results

The effect size (Cohen's $d \approx 0.99$) indicates **a large practically significant effect** in favor of the experimental group. This constitutes direct empirical evidence that education based on the constructed technological model leads to a significantly higher level of mathematical preparation at the end of Grade 7th compared to traditional education. It may therefore be concluded that the developed technological model exerts a substantial positive influence on the development of students' mathematical knowledge and skills in lower secondary education.

3.6.3. Comparative Analysis of the Three-Year Experiment Results

The data clearly demonstrate that education based on the constructed technological model leads to significant and sustainable improvement in student achievement. Differences between groups are statistically significant according to the Welch t-test and the HL estimator, and the effect increases over time. This provides strong evidence of the effectiveness and practical usefulness of the applied methodology.

3.6.4. Longitudinal Analysis of the Three-Year Experiment

Achievement development in the experimental group follows a clearly upward trajectory, whereas in the control group a decline is observed as early as Grade 5th, followed by stabilization at lower values. The difference between groups increases at each subsequent stage, which confirms the results of the statistical tests for *a significant effect of the applied technological model of education*.

3.6.5. Longitudinal Comparative Analysis of the Development of Cognitive Levels (Knowledge, Understanding, Application)

Longitudinal comparison between the two groups unequivocally confirms that differences deepen as education progresses. This supports the conclusion that the observed positive effect in the experimental group results from the purposeful and systematic implementation of the developed technological model of education.

3.7. Conclusions and Generalizations from the Empirical Study

Taken together, the results allow for a scientifically grounded conclusion that **the constructed technological model of mathematics education demonstrates a clearly expressed and practically significant positive effect** on both the dynamics and final level of student achievement in lower secondary education.

CONCLUSIONS FROM CHAPTER THREE

The research presented in Chapter III demonstrates that integrating the technological model for ensuring continuity through a synergetic approach in lower secondary mathematics education yields positive outcomes in several key areas:

- 1) Good organization and methodological consistency – clearly defined goals, participant selection criteria, stages, and measurement tools provide a solid foundation for conducting the experiment. This increases the reliability of the data obtained.
- 2) The implementation of the experiment in a natural learning environment contributes to the validity of the results. Students are trained by applying the constructed technological model for three school years, which allows us to track the sustainability of the effect of the model.
- 3) The didactic materials used in the experiment are highly consistent with the cognitive and age characteristics of the students. They support not only the consolidation of knowledge, but also the development of logical thinking, adaptability, and skills for working in new situations.
- 4) The approbation of the model in a school environment confirms its applicability and effectiveness. Greater engagement and independence on the part of students is reported, as well as a better understanding of the learning content.
- 5) Improvements are observed in test scores on key mathematical topics, especially on tasks requiring logical reasoning, modeling, and the use of problem-solving strategies.
- 6) The analysis of the results unequivocally confirms the hypothesis that education based on the constructed technological model leads students to a higher level of understanding, mastery, and application of mathematical knowledge.

In conclusion, the exposition in Chapter III confirms that implementation of the synergetic model, grounded in modular structure and methodological coherence, represents an effective approach to enhancing the quality of mathematics education in lower secondary school.

CONCLUSION

Within the framework of the study, the hypothesis was formulated and substantiated that the systematic implementation of a technological model based on a synergetic approach in the education of students in Grades 5th – 7th leads to improved quality of mathematical achievement, reduction of persistent learning errors, and a higher level of learning motivation and engagement. The model integrates didactic principles, cognitive mechanisms, and technological solutions, placing emphasis on active, meaningful, and sustainable knowledge acquisition.

The formulated hypothesis has been fully confirmed. The results of the experiment demonstrate that the purposeful and consistent application of the synergetic technological model constitutes a reliable didactic strategy for optimizing the educational process at the lower secondary stage and has the potential for extension to subsequent levels of education.

The study opens new avenues for further research – toward adapting the model for education in other subjects, expanding the system for working with errors, developing digital environments for diagnostics and monitoring, and validating the model in different types of schools.

In conclusion, it may be stated that the proposed technological model represents a significant contribution to the methodology of mathematics education by providing a practical and scientifically substantiated mechanism for ensuring continuity, improving the quality of mathematics education, and developing competencies aligned with the requirements of contemporary education.

Main Contributions of the Dissertation

The primary objectives and tasks formulated in the Introduction of the dissertation have been accomplished. The main contributions of the present dissertation may be characterized as scientific-applied and practice-applied. The relationships between the contributions, the research objectives, their location within the dissertation, and the related publications are presented in *Table 1*.

Table 1. The relationships between the contributions, the research objectives, their location within the dissertation, and the related publications

Contribution	Objectives	Chapter	Publications
Scientific-Applied contributions			
<i>SA1.</i>	1	Chapter I 1.2. ; Chapter II 2.3	2, 3, 4
<i>SA2.</i>	5	Chapter II 2.4.	5
<i>SA3.</i>	2, 3, 4	Chapter I 1.4.; Chapter II 2.1.; 2.5.	1, 3, 4
Practice-Applied contributions			
<i>PA1.</i>	6	Chapter III 3.1.; 3.3.; 3.5.	
<i>PA2.</i>	6	Chapter II 2.5.; Chapter III 3.2.	
<i>PA3.</i>	6	Chapter III 3.6.	
<i>PA4.</i>	6	Chapter II 2.4.1.; Appendix 3. 3.1.	

Scientific-Applied contributions of the dissertation research are:

SA1. A theoretical foundation of the concept of „continuity“ has been developed within the context of the synergetic approach and the significance of continuity for mathematics instruction has been substantiated.

SA2. A technological model has been developed and implemented in mathematics education at the lower secondary stage.

SA3. The curriculum content and teaching methodology in Grades 5th – 7th have been analyzed with regard to continuity, and its actual state has been diagnosed. The possibilities for applying a synergetic approach in a pedagogical context have been examined.

Practice-Applied contributions of the dissertation research are:

PA1. The constructed didactic model has been approbated in a real school environment.

PA2. A didactic toolkit has been developed, including six modules and a system of criteria and indicators for diagnostics.

PA3. Based on analysis and statistical processing of the experimental results, the dissertation hypothesis has been confirmed, and corresponding conclusions and practical recommendations have been formulated.

PA4. Methodological guidelines for teachers have been developed, encouraging purposeful use of the synergetic approach in planning, implementing, and analyzing educational situations.

Recommendations and Prospects for the Development of the Technological Educational Model

The technological educational model developed and approbated in the study demonstrates significant potential for further development. Its flexibility, modular structure, and orientation toward continuity, synergetics, and active learning methods make it suitable both for further refinement and for broader implementation. Prospects for improvement are multidirectional – toward digitalization, interdisciplinarity, enhanced diagnostics, analytical capacity, and teacher training. With systematic development and institutional support, the model may evolve into a sustainable practice of high scientific and practical value.

Publications Related to the Dissertation Topic

1. Karadzhova, P., D. Boykina. (2021). Synergetic Fundamentals of Mathematics Education, In: *Proceedings of the Anniversary International Scientific Conference „REMI 2021“*, 22 – 24 October 2021, pp. 187-193, ISBN: 978-619-202-711-7
2. Karadzhova, P., D. Boykina. (2022). Synergistic Aspects of Continuity in Education, In: *Science and Education a New Dimension*. Pedagogy and Psychology, X (102), Issue: 263, 2022 Febr. <https://doi.org/10.31174/SEND-PP2022-263X102-04>
3. Karadzhova, P., (2022). Continuity in the Use of a Synergetic Approach in Mathematics Education When Working with Word Problems, In: *Student Almanac (9) Faculty of Pedagogy, Thracian University – Stara Zagora, Anniversary International Scientific Conference „Education and Contemporary Challenges“ 24 – 25 June 2022*, pp. 16-21, ISSN 2603-3178
4. Karadzhova, P., (2024). Synergetics in education as a new educational paradigm. *Tereni (8), Seminar for PhD students and young scientists: Academic trajectories. 1 – 4 September 2023. „Rodopi“ Center, Slaveyno, SU „St. Kliment Ohridski“*, pp. 136-145. DOI: [10.60053/TER.2024.8.136-145](https://doi.org/10.60053/TER.2024.8.136-145)
5. Karadzhova, P., (2025). A Technological Model for Providing Synergetic Succession in Mathematics Education. In: *Proceedings of the Anniversary International Scientific Conference “Synergetics and Reflection in Mathematics Education”, October 22-24, 2025, Pamporovo, UP „Paisii Hilendarski“*, pp. 75-82, ISBN: 978-619-7768-41-1.

Presented Papers at International and National Conferences:

1. Report on the topic „Synergetic Fundamentals of Mathematics Education“, Anniversary International Scientific Conference „REMIA 2021“, 22 – 24 October 2021, Plovdiv.
2. Report on the topic „Continuity in the Use of a Synergetic Approach in Mathematics Education When Working with Word Problems“, Anniversary International Scientific Conference „Education and Contemporary Challenges“ 24 – 25 June 2022.
3. Report on the topic „Synergetics in education as a new educational paradigm“, Seminar for PhD students and young scientists: Academic trajectories. 1 – 4 September 2023. „Rodopi“ Center, Slaveyno.
4. Report on the topic „A Technological Model for Providing Synergetic Succession in Mathematics Education“. Anniversary International Scientific Conference “Synergetics and Reflection in Mathematics Education”, October 22-24, 2025, Pamporovo.

Noted Citations

Cited article:

Karadzova, P., (2024). Synergetics in education as a new educational paradigm. *Tereni* (8), *Seminar for PhD students and young scientists: Academic trajectories. 1 – 4 September 2023. „Rodopi“ Center, Slaveyno, SU „St. Kliment Ohridski“, pp. 136-145. DOI:[10.60053/TER.2024.8.136-145](https://doi.org/10.60053/TER.2024.8.136-145)*

From:

1. Momcheva, G., T. Glushkova. (2025). Visualizations: the Synergy of Digital and Mathematical Competencies. *Anniversary International Scientific Conference “Synergetics and Reflection in Mathematics Education”, October 22-24, 2025, Pamporovo, University Press – Paisii Hilendarski, pp. 95-100. ISBN: 978-619-7768-41-1.*
2. Marinov, D., D. Boykina. (2025). Interactive Lesson on the Topic “Derivative of a Function”. *Anniversary International Scientific Conference “Synergetics and Reflection in Mathematics Education”, October 22-24, 2025, Pamporovo, University Press – Paisii Hilendarski, pp. 89-94, ISBN: 978-619-7768-41-1.*

Acknowledgements

I would like to express my sincere gratitude and appreciation to my scientific supervisor, Assoc. Prof. Dobrinka Milusheva-Boykina, PhD, for her support and trust, for the time devoted and the valuable guidance, as well as for her understanding and patience.

I extend my thanks to the members of the Department of „Education in Mathematics, Informatics and Information Technology“ for their responsiveness and constructive recommendations during the work on this dissertation.

I am also deeply grateful to my family for their constant understanding, support, and encouragement.

BIBLIOGRAPHY

- [1] Blakemore, S. J., & Mills, K. L. (2014). Is adolescence a sensitive period for sociocultural processing?. *Annual review of psychology*, 65(1), 187-207.
- [2] Bruner, J.S. (1960). *The process of education: a landmark in educational theory*. Cambridge: Harvard University Press.
- [3] Budanov, V. G. (1996). The era of bifurcations and synergetics in education. – *Moscow Synergetic Forum*. Theses. M., pp. 42.
- [4] Desev, L. (2015). *Synergetics. Introduction and Vocabulary*. Sofia: Ekoprogress, 464 p., ISBN 978-954-2970-37-8.
- [5] Dweck, C. S. (2000). *Self-Theories: Their Role in Motivation, Personality, and Development*. Psychology Press.
- [6] Frenkev, D. G. & V. B. Milushev (2005). On some aspects of continuity in learning to solve word problems. In: „*Some aspects of continuity in mathematics education in primary school* “. Ed. P. D. Petrov, Stara Zagora, Kota Publishing House, pp. 75 – 88.
- [7] Galabova, D. (2012). *Pedagogical Synergetics*. Veliko Tarnovo: UP „St. St. Cyril and Methodius“, 210 p., ISBN: 978-954-524-879-5.
- [8] Garov K., V. Bizova-Laleva. (2013). Geometric Model of Motion Problems Using Dynamic Software, *Science and Education a New Dimension: Pedagogy and Psychology*, Vol.9, Budapest, ISSN 2308-5258.
- [9] Gaydarova, M., T Terzieva, A. Rahnev. (2020). ICT Based Approaches to Increase the Efficiency of the Educational Process. *Anniversary International Scientific Conference “Synergetics and Reflection in Mathematics Education”, October 16-18, 2020, Pamporovo*, UP „Paisii Hilendarski“, ISBN: 978-619-202-595-3.
- [10] Grozdev, S. (2007). *For High Achievements in Mathematics: The Bulgarian Experience (Theory and Practice)*. Sofia: „Ruta“, 295 p., ISBN: 978-954-92139-1-1
- [11] Grozdev, S., B. Kirilova. (2015). Formative Assessment in the Pedagogical Practice of the American College of Sofia, *Didactical Modeling – Volume 5. Report presented on 09.02.2015 at the seminar “Didactical Modeling”, IMI – BAS*, 10 p.
- [12] Haken, H. (1983). *Synergetics: An Introduction: Nonequilibrium Phase Transitions and Self-Organization in Physics, Chemistry and Biology*. Springer, Berlin.
- [13] Inhelder, B., & Piaget, J. (1958). *Adolescent thinking*. In B. Inhelder, J. Piaget & A. Parsons, S. Milgram (Trans.). *The growth of logical thinking: From childhood to adolescence* (pp. 334–350). Basic Books.

- [14] Karadzhova, P., D. Boykina. (2022). Synergistic Aspects of Continuity in Education, In: *Science and Education a New Dimension*. Pedagogy and Psychology, X (102), Issue: 263, 2022 Febr. <https://doi.org/10.31174/SEND-PP2022-263X102-04>.
- [15] Karadzhova, P., (2025). A Technological Model for Providing Synergetic Succession in Mathematics Education. In: *Proceedings of the Anniversary International Scientific Conference "Synergetics and Reflection in Mathematics Education", October 22-24, 2025, Pamporovo, UP „Paisii Hilendarski“, pp. 75-82, ISBN: 978-619-7768-41-1.*
- [16] Knyazeva, E. N., S. P. Kurdyumov. (2002). *Foundations of Synergetic: Exacerbation Regimes, Self-organization, Tempo-worlds*, Moscow: Alethea, 414 p. ISBN 5-89329-517-X.
- [17] Koleva, K. (2021). *Logical problems. Monograph*, Veliko Tarnovo: ITI, 144 p., ISBN: 978-619-7602-05-0.
- [18] Milushev, V. (2010). Elements of the Reflexion and Synergetics in Mathematics Education. In: Anniversary International Scientific Conference "Synergetics and Reflection in Mathematics Education", September 10–12. 2010, Bachinovo, pp. 213-220.
- [19] Milusheva-Boykina, D., & Milushev, V. (2014). Research on the activity of composing educational mathematical problems. In: *Science and Education a New Dimension: Pedagogy and Psychology.–II (9), (19)*, pp. 81-85.
- [20] Piaget, J. (1970). *Science of Education and the Psychology of the Child*. New York: Orion.
- [21] Piaget, J., & Inhelder, B. (2008). *The psychology of the child*. Basic books, 192 p., ISBN 0786725338, 9780786725335.
- [22] Ryan, R. M., & Deci, E. L. (2000). When rewards compete with nature: The undermining of intrinsic motivation and self-regulation. In *Intrinsic and extrinsic motivation* (pp. 13-54). Academic Press.
- [23] Sansone, C., & Harackiewicz, J. M. (Eds.). (2000). *Intrinsic and extrinsic motivation: The search for optimal motivation and performance*. Academic Press.
- [24] Siegler, R. S. (1996). *Emerging Minds: The Process of Change in Children's Thinking*. New York, NY: Oxford University Press.
- [25] Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- [26] Zimmerman, B. J., D. H. Schunk, (2001). *Self-regulated learning and academic achievement: Theoretical perspectives* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum.