UNIVERSITY OF PLOVDIV
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Department of Primary School Pedagogy

Keti Agop Angelova

A Technological Model for Pedagogical Interaction in Forming Basic Quantitative Concepts in 6–7-Year-Old Children

ABSTRACT

of a dissertation for the award of the educational and scientific degree "Doctor"
in Higher Education Field 1. Pedagogical Sciences,
Professional Field 1.3. Methodology of Teaching...,
Doctoral Program: Methodology of Teaching Mathematics

Scientific Supervisor: Associate Professor Dr. Dimitrina Petrova Kapitanova

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The dissertation has been reviewed and approved for defense by the Department of Primary School Pedagogy at the Faculty of Education of "Paisii Hilendarski" University of Plovdiv in an extended session held on October 14, 2025.

The dissertation entitled "*A Technological Model for Pedagogical Interaction in Forming Basic Quantitative Concepts in 6–7-Year-Old Children"* consists of: a title page, table of contents, introduction, three chapters, summary, conclusion, scientific contributions, bibliography, appendices, and a declaration of originality. The total length is 311 pages, of which 199 comprise the main text, 11 are bibliography, and 101 are appendices. A total of 149 sources have been used on the topic.

The public defense of the dissertation will take place on **December 10, 2025**, at **11:00 AM**, in **Seminar Room 405**, **New Building, 4th floor**, at "Paisii Hilendarski" University of Plovdiv.

The defense materials are available for review at the library of the Faculty of Education, "Paisii Hilendarski" University of Plovdiv, located at **236 Bulgaria Blvd., Plovdiv**.

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Structure and Main Content of the Dissertation

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INTRODUCTION

A key stage in a child's development is preschool education, whose main goal is to prepare 6–7-year-old children for academic and social life at school. During this period, children develop key skills and competencies, such as:

– cognitive;
– social, emotional, and communicative;
– linguistic;
– mathematical;
– and others.

Cognitive skills are a set of mental abilities that a person uses to acquire new knowledge, perceive and process incoming information, reason about arising issues, and solve problems.

A crucial role in the mental development of preschool children is played by the acquisition of basic mathematical concepts. These actively influence the formation of activities that are essential for understanding the world around them.

State educational standards in the area of children's cognitive development include the formation of initial ideas about the properties and relationships of objects in their surrounding world, such as: shape, color, size, quantity, number, part and whole, space, and time. The formation of initial ideas about these concepts forms the basis of elementary mathematical competence in preschool children.

Recognizing the importance of good preliminary mathematical preparation for preschool children, many educators and methodologists have developed various educational models and share best practices that provide children with the necessary stimuli and support.

For the educational needs of modern children, a teacher is required: to be able to restructure the content and forms of work with children; to exert creative efforts; to seek new approaches for each child, taking into account their level of development; to have the necessary life experience; to possess the ability to present knowledge and skills in an accessible way for children; to actively use innovations and scientific achievements in the field of pedagogy and methodology of teaching mathematics in preschool education.

The methodology for forming basic mathematical concepts in preschool children has gone through a long developmental path. During the

17th to 19th centuries, issues related to the content and methods of teaching mathematics (elements of elementary arithmetic) to preschool children and the formation of concepts of quantity, number, size, measures of measurement, time, and space were reflected in advanced pedagogical education systems developed by J. A. Comenius (1658), J. H. Pestalozzi (1803-1805), K. D. Ushinsky (1864-1870), and others.

In the rapidly changing realities of the modern world, the need arises for a new concept of preschool education, which defines new educational goals and policies.

The relevance of the research problem is determined by practical needs. Preschool mathematical education requires an innovative, interdisciplinary technological model of pedagogical interaction that maximally meets the current needs and possibilities of each child and the demands of society as a whole.

The motives for choosing this dissertation research are:

– the desire to conduct a comprehensive study related to the elementary mathematical competence of 6–7-year-old children;

– upcoming innovations in mathematical education;

– scientific interest in creating and experimenting with a technological model for pedagogical interaction in forming elementary quantitative concepts in 6–7-year-old children.

The theoretical significance of the research lies primarily in the in-depth study of the theoretical and practical importance of the problem, clarifying some specific aspects of the educational process for children in preparatory groups during the formation of knowledge, skills, and competencies from the core of quantitative relations, and defining and using specific criteria and indicators for assessment, according to the needs of this dissertation work:

1. Revealing the possibilities offered by the interdisciplinary approach to enhancing the intellectual and mathematical readiness of 6–7-year-old children for school;

2. Developing an interactive technological model for pedagogical interaction in forming elementary quantitative concepts for future students;

3. Developing a methodological system of cognitive tasks and games and the accompanying methodological instructions for educators.

The practical significance of the work consists in the possibility of its use by current and future preschool teachers in the preparation and implementation of pedagogical interaction processes in mathematics with children in the preparatory group of kindergarten.

As an experimental base, kindergartens in Plovdiv, Haskovo, and Sofia were used.

CHAPTER ONE

THEORETICAL FRAMEWORK OF THE PROBLEM

1.1. Didactic Aspects of Mathematics Education in Kindergarten

1.1.1. Pedagogical Concepts and Systems for Organizing Preschool Education by Maria Montessori, Rudolf Steiner, Célestin Freinet, and Loris Malaguzzi

The possibilities for improving the process of forming mathematical concepts in preschool children are not yet fully exhausted, as the limits and potential abilities of children to master mathematical ideas remain largely unexplored.

One of the pedagogical systems that holds a significant place in preschool education is that of Maria Montessori.

Elements of Maria Montessori's pedagogical technology can be applied in all educational situations, enabling teachers to enhance children's ability to grasp mathematical concepts and elevate the overall cognitive activity of the children to a higher developmental level.

Thanks to their clarity and concreteness, the didactic materials proposed by Montessori allow children to understand difficult abstract mathematical concepts and operations.

Rudolf Steiner was an Austrian philosopher and esotericist. At the beginning of the 20th century, Steiner attempted to impose his views in the field of pedagogy, preschool, and school education.

Steiner first outlined his own educational concept in detail in his "Course on Popular Pedagogy." The school of the future should be built on a profound understanding of the human being. According to Steiner's pedagogical ideology (1919), teachers are individuals in whom pedagogy awakens through an understanding of human nature, and whose interests are not limited only to educational practices and teaching.

Steiner regarded pedagogy as a science of the human being, where the human is considered in three aspects: physical, soul, and spiritual essence—a scientifically grounded humanistic concept that awakens love for humanity.

School curricula and teaching methods are developed to respond as best as possible to changing needs. Rudolf Steiner did not aim to provide his readers with a set of "pedagogical recipes." On the contrary, he demonstrated that pedagogical work does not tolerate stereotypes; they kill it.

Education is an art, and the educator, practicing it based on precise knowledge, must use entirely individual methods personal to them. His pedagogy can be seen as a path of self-education for every teacher.

Célestin Freinet was a French pedagogue and reformer. He developed his own method for teaching and educating children and opened his own school at the age of 24.

Freinet viewed the spontaneous needs and interests of the child as objective indicators of "life potential," a phenomenon he incorporated into the organism as a force driving self-development. According to this "life potential," the individual adapts to the environment, functions, and develops.

At the core of Freinet's work are the ideas of reform pedagogy. However, the teacher's own critical view of the nature and development of the child defines the characteristics of his theoretical views, revealing the essence of the anthropological-pedagogical approach to education and training processes.

Loris Malaguzzi (1920–1994) was an Italian educator and psychologist. He vividly demonstrated that every child naturally has "a hundred languages" to express themselves. This is not just ordinary speech but also creativity—painting, sculpture, theater, and many others. However, school often "takes away" 99 of these languages, leaving only one—speech.

The adult's task is to inspire the child towards activity, to develop all the "languages" with which nature has endowed them, to stimulate curiosity, help them understand the world, and find their place in it.

Loris Malaguzzi is the creator of the so-called "Reggio Emilia pedagogy." He prioritized not the early ability to read and count but the development of creative thinking. In the modern world, creative thinking comes to the forefront. This is exactly what pedagogy actively develops by stimulating project-based thinking. Since a project includes various simultaneous activities, it develops imagination and creative thinking.

1.1.2. Goal, Tasks, Functions, and Importance of Modern Preschool Education

In the Republic of Bulgaria, care and education for children in early age are provided in nurseries, kindergartens, and schools that ensure conditions for early child development and preparation for school.

The goal of preschool education (PE) is to provide upbringing, socialization, education, and care for children before they enter the 1st grade, through acquiring a set of competencies—knowledge, skills, and attitudes necessary for the successful transition to school education, and by supporting personal development. PE lays the foundation for lifelong learning by ensuring the physical, cognitive, linguistic, moral-spiritual, social, emotional, and creative development of children, considering the importance of play in the pedagogical interaction process.

The main functions of the educational system are twofold—cognitive and formative. The cognitive function describes the role of education in

transmitting knowledge and developing the cognitive abilities and interests of young learners. The formative function reflects the importance of education in fostering the upbringing of young learners, affirming their life goals, self-awareness, and self-determination as individuals.

By Regulation No. 5 dated 03.06.2016 on preschool education, issued by the Minister of Education and Science, the state educational standard was adopted. It represents a set of requirements for learning outcomes and defines the educational fields through which preschool education is implemented, as well as the goals and content of each educational field, and the requirements for learning outcomes for each educational field in different age groups.

The National Qualifications Framework is developed based on the European Qualifications Framework, adopted by the Council of Ministers, and contains competencies as learning outcomes by levels, corresponding to the stages and degrees of education.

1.2. Modern Pedagogical Technologies and Teaching Models in Preschool Age

*** Modern Pedagogical Technologies**

Modern preschool pedagogy widely uses the rich accumulated experience, revealing the continuity of modern education and pedagogical traditions.

Mathematics occupies a special place in the intellectual development of children, whose level should be determined by the qualitative characteristics of the mastered elementary mathematical concepts.

Modern technologies for the mathematical development of preschool children aim to activate the child's cognitive activity and mastery of the relationships and dependencies of the child on objects and phenomena in the surrounding world.

*** Modern Models of Teaching in Preschool Age**

From 2000 to the present, a modern stage has been observed in the development of the problem of forming elementary mathematical concepts in preschool children. One of the main theoretical principles underlying the formation of elementary mathematical concepts in preschool children in modern preschool institutions is the understanding of the properties and relationships of objects based on practical comparison of sizes through identifying common characteristics in objects—length, width, height (Galperin, V. Davidov, A. Leushina, et al.).

1.3 Theoretical Models for Forming Quantitative Concepts in Preschool Age

1.3.1 Classical Models for Forming Quantitative Concepts

The formation of quantitative concepts is fundamental to the development of mathematical competence. In the formation of methodology for developing elementary mathematical concepts during the 19th and early 20th centuries, two directions emerged: one related to the so-called method for studying numbers or the monographic method, and the other related to the method for studying operations, called the computational method.

In the process of forming numerical concepts, work on activating children's vocabulary is very important. Preschool children learn to agree numerals with nouns in gender and number. The teacher pays attention to the fact that we name numbers differently depending on what we are counting.

1.3.2 Pedagogical Concepts of A. M. Leushina, A. A. Stolyar, and L. S. Metlina on Forming Concepts of Quantitative Relations

Analyzing methods for forming quantitative concepts in preschool children, we can conclude that all authors, when constructing their methods, rely on the basic method proposed by A. M. Leushina. They emphasize common tasks, but at the same time, E. I. Shcherbakova and A. A. Stolyar significantly expand the content of their methods.

The general goal of the methodology is to study and develop the didactic foundations of the process of forming elementary mathematical concepts in preschool children. The methods proposed by A. A. Stolyar contain the basic component introduced by A. Leushina as well as new practical and verbal methods.

The full development of quantitative concepts in a child in the preparatory group of kindergarten has a huge influence on the overall development of their personality, prepares the child for school education, and broadens their life opportunities.

1.4 Educational Direction "Mathematics" in Preschool Education. Core "Quantitative Relations"

The specificity of the educational direction "Mathematics" lies in the fact that, on the one hand, its content must ensure the formation of elementary concepts of basic mathematical notions that are later studied at school, and on the other hand, mathematical training stimulates general cognitive activity and develops the child's mental abilities, which are the foundation for their intellectual development.

The formation of elementary mathematical concepts is a purposeful process of transmitting and assimilating knowledge, techniques, and methods

of mental activity as prescribed by the mathematics program for preschool education.

Mathematical education of preschool children is a purposeful process of pedagogical interaction in which elementary mathematical competencies and methods for understanding mathematical reality are mastered.

1.4.1 Features of Forming Quantitative Concepts and Relations in 6-7-Year-Old Children

The quantitative set theory involves a set-theoretic approach, which plays a major role in studying natural numbers. This approach's primary notions for constructing the arithmetic of natural numbers are sets and one-to-one correspondences (bijections) between sets.

The acquisition of counting skills by children is a complex process, whose origin lies in small children's manipulations with objects. At the same time, digits serve as a kind of accompaniment to children's general movements. Both in chaotic naming of numbers and in naming them in order, children develop auditory-speech-motor connections that determine further mastery of counting.

1.4.2 Forming Concepts of Sets. Set Operations. Counting.

*** Sets. Set Operations.**

The founder of set theory is Georg Cantor, and the concept of a set is one of the first concepts to arise in mathematics. A set is any union into a whole of certain well-distinguishable objects of our perception or thought. The objects that make up a set are called its elements. Any collection of objects forms a finite or infinite set. If it is finite, its numerical value can be enumerated and recorded.

From early childhood, a child is surrounded by objects differing in size, shape, color, and quantity. As the child develops, their relations with the surrounding world change. The child enters the world of objects and phenomena and perceives not only objects but also sounds and movements through the senses. The child develops chaotic, unordered concepts of quantity.

*** Methodical Variants for Working with Sets in Preschool Age:**

1. Assigning sets;
2. Adding or removing objects from a group;
3. Defining a subset – dividing the formed set into subsets – classes;
4. Intersection of two sets: containing elements belonging simultaneously to both sets;
5. Union of two sets;
6. Relations between sets: "more," "less," "as many as";
7. Adding or removing elements to establish the relation "equal";

8. Establishing the independence of quantitative equality between two groups of objects from size and arrangement of the objects.

To succeed in working with sets, it is necessary to know the abilities of preschool children, the dynamics of their joint perception of objects, and their capacity to perform certain generalizing activities.

*** Counting**

Counting activity consists of the ability to practically establish a one-to-one correspondence between elements of two groups and determine their equality or inequality without naming the number.

Counting is the first and most important step and is fundamental in developing mathematical skills. It is very important for children to understand the principle of counting and its role in various life areas from an early age—to understand why counting makes sense. It underlies the development of the child's abstract thinking and their concept of numbers. When entering the world of numbers, children understand that numbers are used in different contexts.

Throughout the preparatory year in kindergarten, children learn to count up to ten in direct and reverse order, use numbers to determine the quantitative characteristics of object sets and the position of each element in an ordered set, group objects by given properties, and name the total number of objects in a group. To form the concept of number, children need to see, hear, touch, and compare sets of various objects, sounds, and movements at an early age.

1.4.3 Forming Concepts of Natural Numbers. Addition and Subtraction Operations with Natural Numbers up to 10

The concept of a natural number is one of the fundamental concepts in mathematics. It expresses the quantity and quantitative relations of things in reality.

Digits are signs (symbols) used to denote numbers. Natural numbers arise during the counting of individual objects (1, 2, 3, etc.) or measurement. They are often called counting numbers. Numbers are a basic component of children’s mathematical development. Work on developing the concept of number starts in preschool and continues into primary education.

Knowing numbers and mastering operations in understanding quantities, children transition from direct methods (application, imposition, visual comparison) to indirect comparison (through measurement with a conventional measure and auxiliary object). Numbers express magnitude and quantity. Performing operations with numbers, which are indicators of magnitudes and quantities of objects in the surrounding world, by applying comparison operations, decreasing and increasing them, conclusions can be drawn about the final quantitative result following the applied actions. The

essence of the number and performing operations with numbers is understood by preschool children over a long period.

1.4.4 Didactic Materials and Games in Mathematics Education in Preschool Age

*** Operational Didactic Material**

Didactics pays special attention to teaching aids and their influence on learning outcomes. Visual aids can be a means of posing or solving a task.

It is necessary to distinguish between the concepts of "visual aids" and "didactic means." Didactic means is a broader term including sets of objects, phenomena, signs, models, actions, and words.

The main goal of using didactic materials is to teach children how to solve logical tasks involving classification by properties, familiarize them with geometric shapes and object forms, sizes, develop basic skills of algorithmic thinking culture, and enhance cognitive processes such as perception, memory, attention, imagination, and foster creativity.

Didactic means are tools for the teacher's work and instruments for children's cognitive activity. Teaching aids are sources of information and one of the most important components of the methodological system.

*** Didactic Games for Developing Elementary Quantitative Concepts**

Mathematical didactic games are among the best-known and most frequently used in modern preschool education as entertaining mathematical materials. These are games specifically designed to achieve educational outcomes. Their specific feature is intentionality, with a clear goal and expected result. Their main purpose is to teach children to distinguish, highlight, and name various objects, numbers, geometric shapes, directions, etc.

*** Educational Literature for Preschool Children**

The first children's books with mathematical content were created by analogy with school textbooks. This allows precise definition of the content mastered by preschool children and outlines its gradual complication, facilitating the learning and development process.

Regarding children's mathematical development, the value of educational books lies in the special clarity of the content conveyed. A book is a synthesis of arts (literature, graphics, printing), and the images it presents are perceived by children as a unity of various expressive means (words and illustrations). These means reinforce each other, contributing to a richer, "enhanced" image that is easier to understand.

1.5 Interdisciplinarity, Integrativity, and Project-Based Learning in Preschool Education

*** Interdisciplinarity in Preschool Education**

The changing role of knowledge in society requires restructuring the educational system based on interdisciplinary integration of various fields of knowledge. The ability to use different types of knowledge in solving practical and theoretical problems becomes the main method of interaction.

Interdisciplinarity in education refers to when a child receives knowledge on a topic or problem from multiple scientific perspectives.

Applying an interdisciplinary approach occurs when teachers modify parts of the curriculum from two or more educational areas suitable for a specific problem or topic.

*** Integrativity in Preschool Education**

The process of integration (from Latin integratio – connection, restoration) is the unification into a whole of previously separate parts and elements of a system based on their interdependence and mutual complementation.

Integration is a complex interdisciplinary scientific concept used in several humanities: philosophy, sociology, psychology, pedagogy, etc. Research on inter-subject connections shows that this concept becomes a leading category in pedagogical science, whose essential foundation is its connecting, unifying function.

Inter-subject connections are defined as a multifunctional pedagogical category denoting synthesizing, integrative connections between objects, phenomena, and processes of reality, reflected in the content, forms, and methods of teaching and learning processes in educational institutions.

*** Project-Based Learning in Preschool Education**

The project-based approach is naturally and harmoniously integrated into the educational process of kindergarten. The effectiveness of using the "project method" in the education and upbringing of preschool children is multifaceted.

Applying the project-based approach in pedagogical interaction allows teachers, children, and parents to unite, learn to work as a team, collaborate, and plan their work. Each child can express themselves, feel needed, and accordingly gain confidence in their abilities.

1.6 Conclusions to the First Chapter

The analysis of scientific and methodological literature on the dissertation topic allows us to draw the following conclusions:

– The age of 6–7 years is favorable for forming quantitative concepts and relations;

– An effective means of teaching preschool children is didactic games. Using games increases the efficiency of the educational process. They contribute to the development of memory and logical thinking in children, having a huge impact on their overall intellectual development;

– Applying an integrated approach in the process of pedagogical interaction in mathematics helps children gain a holistic picture of the world, enables them to realize their creative abilities, develops their social skills, and contributes to the free sharing of their impressions;

– Building integrated innovative technologies in kindergarten is related to providing a good information-educational environment for the modern child. This raises the issue for the current teacher of improving and modernizing the educational process, and of wider use of new, more rational technologies.

CHAPTER TWO

DESIGN OF THE DISSERTATION RESEARCH

2.1. Problem Statement

The formation of basic mathematical concepts is a key aspect of the cognitive development of preschool-aged children. Mastering mathematical skills at an early age is a significant milestone that influences the overall intellectual development of young learners.

Mathematical concepts form the foundation upon which children build their understanding of the world around them. They are important not only for the development of mathematical skills but also for enhancing general problem-solving abilities, logical and critical thinking. Children who acquire knowledge and skills related to fundamental mathematical notions at an early age are better equipped to face the challenges of formal education in the later stages of their academic journey. Moreover, early exposure to mathematics stimulates the development of language skills, spatial thinking, and even social competencies, as it often involves collaboration and communication with peers.

Early mathematics education fosters logical thinking, problem-solving abilities, and cognitive flexibility in children.

The development of deep and lasting mathematical competencies, particularly in the domain of quantitative relationships, is a gradual and complex process that unfolds progressively throughout the preschool years.

2.2. Significance of the Problem

The formation of basic quantitative concepts in 6–7-year-old children is of fundamental importance for their cognitive and intellectual development. This age marks a transitional period during which children begin to establish the foundations of mathematical literacy that will support them throughout their lives.Within the context of addressing quantitative concepts in 6–7-year-olds, STEM education can be effectively promoted through a rich variety of practical and interdisciplinary approaches. These approaches integrate all educational domains studied in preschool, creating a comprehensive and enriched learning environment. Incorporating sciences such as technology, engineering, and mathematics into early childhood education offers opportunities for developing problem-solving skills, creativity, and innovative thinking.

Mathematics is a key component of the early childhood education curriculum, and quantitative concepts lie at the core of foundational mathematical education.

The learning environment also plays a crucial role in children's development. Well-organized spaces in kindergartens and thematic centers for various activities provide opportunities for children to participate freely and naturally in different types of experiences. The creation of educational zones where children can explore and experiment stimulates their curiosity and desire to learn.

2.3. Object and Subject of the Research

2.3.1. Object of the Research

The object of the present research is the formation of basic concepts of quantitative relationships in mathematics among children aged 6–7.

2.3.2. Subject of the Research

The subject of the research is the process of pedagogical interaction in the formation of basic quantitative concepts through a new interactive technological model when working with 6–7-year-old children.

2.4. Aim, Tasks, and Hypotheses of the Research

2.4.1. Aim of the Research

Aim 1: To develop and pilot an interactive technological model of pedagogical interaction for the formation of basic quantitative concepts during the education of 6–7-year-old children.

Aim 2: To create an innovative approach for assessing the intellectual and mathematical readiness of 6–7-year-old children for school.

2.4.2. Research Tasks

To achieve the set goals, the study involves completing the following tasks:

1. Theoretical exploration of the researched problem.

2. Conducting a pedagogical experiment with 6–7-year-old children to assess the level of formation of basic concepts of quantitative relationships.

3. Development of an interactive technological model of pedagogical interaction with 6–7-year-old children for forming basic quantitative concepts.

4. Creation of a toolkit with a criterion-based scale for assessing the results of the conducted research.

5. Summarizing and analyzing the obtained results. Determining the effectiveness of the interactive model.

2.4.3. Research Hypotheses

1. **Null Hypothesis (H_0)**:

It is assumed that there is no significant difference in the level of formation of basic quantitative concepts between children educated using the interactive model and those taught using traditional methods. This means that the interactive model does not have a substantial effect on the children's results.

2. **Main Hypothesis (H_1)**:

It is assumed that through pedagogical interaction with 6–7-year-old children, the application of an interactive technological model will improve the formation of basic quantitative concepts in mathematics.

This means that children taught using this model will demonstrate higher results in acquiring quantitative concepts compared to those educated through traditional methods.

3. **Additional Hypothesis (H_2)**:

It is assumed that the implementation of an interdisciplinary approach will enhance the intellectual and mathematical readiness of children in preparatory groups for the first grade.

Children are expected to develop better cognitive skills, increased motivation for learning, and greater self-confidence, which will facilitate their transition to the school environment.

2.5. Research Methods

To achieve the set objectives, accomplish the goals, and validate the hypotheses formulated in this dissertation, a variety of research methods were used in the experimental study. These methods were selected based on the specific nature of the research problem and the age characteristics of preschool children. The methods can be grouped into the following categories:

2.5.1. Empirical Methods:

1. **Theoretical Study –** An extensive theoretical review of current specialized literature related to the research problem of the dissertation was

conducted. The aim was to uncover the conceptual characteristics of the interactive technological model of pedagogical interaction in the process of developing basic quantitative concepts in 6–7-year-old children.

*** Survey No. 1** – This method was used to investigate the opinions of professionals working in the field of preschool education regarding the current state, trends, and challenges in forming basic mathematical concepts in 6–7-year-old children.

*** Survey No. 2** – The repeated use of the survey method aimed to confirm or reject the additional hypothesis proposed in this study, by gathering the opinions of kindergarten teachers. The survey focused on collecting information about:

– the application of interdisciplinary educational approaches in practice;

– observed effects on cognitive development, including mathematical thinking and literacy in 6–7-year-old children;

– levels of motivation, self-confidence, and adaptability of children during their transition to the school environment.

*** Observation** – One of the most commonly used empirical methods in pedagogical research. In the context of the present study, observations were conducted in real educational situations to record the natural behavior of the children and the effectiveness of the applied pedagogical methods. Observations focused on reactions, engagement, interaction between children and teacher, and the degree of mastery of quantitative concepts. The observation was structured according to pre-defined criteria, allowing for the systematization and analysis of collected data.

*** Demonstration** – This method is widely used in working with children in kindergartens. Also known as the illustrative-demonstrative method, it allows for continuous visual contact between observers and the visual representation. In this study, demonstrations related to quantitative relationships were conducted using various visual and manipulative tools such as illustrations, images, pictures, 3D models, interactive whiteboards, etc.

*** Explanation** – As a method used with preschool children, explanation complements the demonstration method.

*** Conversation** (Dialogue Method) – Widely applied in work with children. Conversation as a method is linked to dialogical exploration of the educational content. In this study, it was used for discussing quantitative concepts, solving problem situations, and exchanging ideas among children.

*** Storytelling** – As a pedagogical method, storytelling has a dual function. On the one hand, it can be used as a motivator, and on the other, it

can be a story composed by the children themselves, serving as a diagnostic tool for assessing knowledge gained through pedagogical interaction.

*** Exercise** – While typically used in school education, exercises can be adapted for preschool age. In this study, exercises were applied in the form of games and practical tasks, encouraging children's active participation and interest.

*** Play** – The fundamental and leading method for organizing all pedagogical situations with preschool children. Play is applicable in all areas of the educational environment and provides a natural and motivating context for learning and development.

*** Didactic Experiment** – This is the primary empirical research method in the present study. The didactic experiment involves planned pedagogical interventions using specific methods and tools to test their effectiveness. The experiment was conducted in three stages:

– **Stating (Diagnostic) Stage** – Initial assessment of the level of development of quantitative concepts in 6–7-year-old children through tests, observations, and conversations to determine the starting point of the participants;

– **Formative Stage** – An interactive technological model of pedagogical interaction was implemented with the children in the experimental groups. During this stage, children participated in specially designed educational situations involving a wide range of interactive methods, games, and digital technologies;

– **Control Stage** – A final assessment was conducted to measure the changes and effectiveness of the applied methods. The results were compared with those from the stating stage to determine the degree of progress.

*** Expert Evaluation** – Analysis and assessment of the impact on the development of basic quantitative mathematical concepts in 6–7-year-old children. Expert evaluations were based on the opinions of specialists in preschool education, contributing to the objectivity and validation of the study results.

*** Testing** – This method was used with children to assess results from the initial and final diagnostics. The tests were developed according to the age characteristics of the children and aimed at evaluating the mastery of quantitative concepts.

*** Statistical Methods for Processing, Systematizing, and Summarizing Empirical Data** – To analyze the collected data, statistical methods were applied, enabling objective evaluation of the research results. Statistical software was used for data processing, including methods such as mean value, standard deviation, independent and dependent t-tests, among others. This allowed for the determination of the statistical significance of

differences between the control and experimental groups and the formulation of evidence-based conclusions about the effectiveness of the applied interactive model.

2.5.2. Additional Methods:

*** Comparative Analysis –** Used to compare results between different groups and research stages. This helps identify trends and patterns in the data.

*** Content Analysis –** Applied to the analysis of qualitative data, such as responses from surveys, observations, and conversations. It allows for the extraction of meaningful information and interpretation of results in the context of the research problem.

Statistical Methods Used for Analysis:

1. Descriptive Statistics – Description of results using frequency distribution, diagrams, and statistical values. It focuses only on known, accessible data that can be realistically recorded and used.

2. Cronbach's Alpha Reliability Test – A method for assessing reliability. It provides a reliable quantitative comparison of a shared variable or covariance among numerous items that make up a given instrument measuring a common variable.

3. Mann-Whitney U Test for Two Independent Samples – A non-parametric method used to compare two samples and test the hypothesis.

– Wilcoxon Signed-Rank Test for Two Related Samples – A method for analysis and statistics that supports the hypothesis and demonstrates the positive effect of innovative pedagogical activity. The application of the Wilcoxon signed-rank test ensures systematicity, variability, and flexibility in the planning and implementation of the interactive educational process.

Applied Software:

To process the data and conduct statistical analyses, the specialized software Microsoft Excel was used, with additional built-in statistical functions.

2.6. Research Tools

2.6.1. Design of the Stating (Diagnostic) Experiment

The purpose of the stating experiment is to assess the level of knowledge and skills acquired by 6–7-year-old children regarding basic quantitative relationships in mathematics and to determine whether there are differences in the level of acquired competencies between the children in the experimental and control groups. To achieve this aim, the following tasks were formulated:

1. To explore the difficulties encountered by educational professionals in the process of developing mathematical concepts in 6–7-year-old children.

2. To determine the level of knowledge and skills related to basic quantitative relationships in mathematics among 6–7-year-old children.

3. To assess the children's achievements resulting from the experimental study by defining the following specific indicators:

– **First indicator:** Working with object sets.

– **Second indicator:** Comparing the number of objects (up to ten) between two sets.

– **Third indicator:** Ordering the sequence of numbers up to ten.

– **Fourth indicator:** Counting to 10 in ascending and descending order; counting objects up to ten.

– **Fifth indicator:** Determining the position of objects in a sequence of ten items.

– **Sixth indicator:** Identifying the number of objects (up to ten) and matching them with the corresponding digit.

– **Seventh indicator:** Solving simple arithmetic problems involving a single operation.

4. To define evaluation criteria for each child's achievement based on the above indicators:

– **High level (3 points)**: The child completes the task independently;

– **Average level (2 points)**: The child completes the task with some assistance;

– **Low level (1 point)**: The child is unable to complete the task.

2.6.2. Design of the Formative Experiment

At this stage of the dissertation, a specially developed **innovative technological model for pedagogical interaction** aimed at developing basic quantitative concepts in 6–7-year-old children is presented. The goal of the formative stage is to enhance children's knowledge and skills in solving cognitive tasks and to improve their competencies related to the formation of basic quantitative concepts in preschool (preparatory group) education.

To achieve the model's objectives, 15 diverse **interdisciplinary pedagogical situations** were developed under the thematic title "*Mathematics through Stories and Creativity"*, distributed across three separate modules.

2.6.3. Design of the Control Experiment

The goal of the control experiment is to assess the **effectiveness of the technological model** for pedagogical interaction in developing basic quantitative concepts in 6–7-year-old children, as implemented during the formative stage. By comparing the results between the experimental and control groups, the degree of influence of the interactive model on the children's cognitive development was determined.

The control experiment is a **key stage** of the study, providing empirical data on the effectiveness of the developed technological model. Through systematic **observation, testing, and movement-based didactic games,** objective information was collected on children's achievements. The analysis of results and confirmation of the hypotheses proposed in the dissertation contribute to the enrichment of pedagogical practice and support the work of preschool educators with young children.

2.7. Technological Model for Pedagogical Interaction in Forming Basic Quantitative Concepts in 6–7-Year-Old Children

2.7.1. Specifics of Model Implementation in the Formative Experiment

The innovative technological model for pedagogical interaction developed in this dissertation is structured into **three distinct modules**, each representing complementary but methodologically different forms of instruction:

*** Module 1: Interdisciplinary Technological Model** - Includes 8 pedagogical situations. In each, children are introduced to a literary work—a fairy tale. Based on the content of each tale, six cognitive tasks are formulated and included in a corresponding worksheet.

*** Module 2: Didactic Movement-Based Games** - Includes 2 pedagogical situations comprising a total of 10 movement-based didactic games with a mathematical focus.

*** Module 3: Interdisciplinary ICT Technological Model** - Includes 5 innovative pedagogical situations conducted with the use of an **interactive whiteboard** and additional **creative engagement** from the children.

A total of **244 children** aged 6–7 from various cities in Bulgaria participated in the formative experiment (experimental and control groups), distributed as follows:

– **Plovdiv** – 180 children
– **Haskovo** – 26 children
– **Sofia** – 38 children

The experimental work was carried out with **122 children** (experimental group) over the course of one academic year, from **September 2023 to May 2024.** During this period, a total of **17 pedagogical situations** were organized and conducted.

CHAPTER THREE

RESULTS FROM THE EXPERIMENTAL STUDY

The organization and implementation of the pedagogical experiment, as well as the testing of the developed innovative technological model, took place from **September 2023 to May 2024**. A total of **48 teachers** and **244 children** from various cities across the country voluntarily participated in the study.

3.1. Results from the Survey Conducted with Educational Specialists – SURVEY FORM No. 1

Before conducting the pedagogical experiment for this dissertation, a survey was carried out among the teaching staff of **16 kindergartens**. The aim of the survey was to explore the opinions of **48 educational specialists** regarding **mathematics education for children in the preparatory group** of preschool.

The survey form consisted of **19 questions**, of which **55% (10 questions)** were **multiple-choice and 45% (9 questions)** were **open-ended**. Each multiple-choice question offered various answer options, with respondents asked to select only one. The open-ended questions allowed participants to express their opinions and provide recommendations concerning the mathematics education of children in the preparatory group and their readiness for school.

The results of the survey indicate a **positive attitude and outlook among pedagogical professionals** towards the introduction of a new **innovative technological model** for pedagogical interaction in mathematics within kindergartens. The willingness of some teachers to take part in the **pilot implementation of the proposed model** through participation in the experimental activities confirmed our initial conclusions drawn from direct observation of current pedagogical practices. This, in turn, provided **further motivation** for our work in developing a new **interactive technological model** for pedagogical interaction with 6–7-year-old children aimed at forming **basic quantitative concepts**.

3.2. Comparative Qualitative and Quantitative Analysis of the Results from the Stating Stage (Stage 1)

At this stage, an **initial diagnostic assessment** was conducted with the aim of determining the level of development of **basic quantitative concepts** among children in the **third age group (5–6 years old)**.

The following activities were carried out:

1. **Criterion-based entry test** – *Worksheet 1* was administered to all children participating in the experiment;

2. **Observations of pedagogical practice** (on-site observations);

3. **Interviews** with kindergarten directors, teachers, and parents regarding the upcoming pedagogical experiment.

A total of **244 children** participated in the experimental study. They were equally divided into two main groups:

*** Experimental group (EG)** – 122 children;

*** Control group (CG)** – 122 children.

All children attended state-funded full-day kindergartens in the following locations:

1. Plovdiv – 180 children

* Kindergarten "*Chuchuliga"* – 60 children

* Kindergarten "*Daga"* – 60 children

* Kindergarten "*Perunika"* – 30 children

* Kindergarten "*Breza"* – 30 children

2. Haskovo – 26 children

** Kindergarten of Arts* – 26 children

3. Sofia – 38 children

* Kindergarten "*126 Tintjava"* – 38 children

A **reliability check** of the results for **Criteria 1 to 7** was performed using **Cronbach's Alpha** for Stage 1 of the pedagogical experiment. A reliability coefficient of **0.6** was obtained, which is interpreted as **acceptable reliability** for the practical goals of the study. Therefore, the results can be used for further analysis. The analysis includes data from all **244 children** across both the experimental and control groups.

To compare the results from Stage 1 between the two groups (experimental and control), the Mann-Whitney U test for two independent samples was applied. The assumptions for using this method were met: the data are ordinal, and the grouping variable has two categories – experimental and control group. The results are presented in the table 1.

The analysis of the data shows that the **mathematical knowledge of the children in both groups is equal** at this stage of the study. There is no visible advantage in the preparedness of 6–7-year-old children in either the **experimental** or the **control group** during the **initial diagnostics**. This suggests that if, at **Stage 2 (the formative stage)**, a **difference in favor of the experimental group** is observed, it can be attributed to the **interactive technological model for pedagogical interaction** presented in this dissertation, aimed at the formation of **basic quantitative concepts** in 6–7-year-old children.

Table 1

Criterion	Group	Count	Average Rank	Sum of Ranks
Criterii 1	EG	122	128,26	15647,50
	CG	122	116,74	14242,50
Criterii 2	EG	122	137,98	16834,00
	CG	122	107,02	13056,00
Criterii 3	EG	122	129,86	20683,50
	CG	122	115,14	9206,50
Criterii 4	EG	122	124,18	15149,50
	CG	122	120,82	14740,50
Criterii 5	EG	122	130,72	15947,50
	CG	122	114,28	13942,50
Criterii 6	EG	122	130,26	15892,00
	CG	122	114,74	13998,00
Criterii 7	EG	122	124,08	15138,00
	CG	122	120,92	14752,00

3.3. Comparative Qualitative and Quantitative Analysis of the Results from the Formative Stage (Stage 2)

Formative Stage – During this stage, **an interactive technological model of pedagogical interaction** was implemented. Children participated in specially designed learning situations that included a wide variety of **interactive methods, games,** and **digital technologies**.

A **reliability check** of the results for **Criteria 1 to 7** was conducted using **Cronbach's Alpha** for Stage 2. A reliability coefficient of **0.8** was obtained, which is interpreted as very **good reliability** for the practical objectives of the study. Therefore, the obtained results can be used for further analysis.

To compare the results of the two groups in **Stage 2,** the **Mann–Whitney U test for two independent samples** was used. The results are presented in the following table:

Table 3

Ranks				
Criterion	**Group**	**Count**	**Average Rank**	**Sum of Ranks**
criterii1	EG	122	140,50	17141,00
	CG	122	104,50	12749,00
	Total	244		
criterii2	EG	122	138,50	16897,00
	CG	122	106,50	12993,00
	Total	244		
criterii3	EG	122	132,50	16165,00
	CG	122	112,50	13725,00
	Total	244		
criterii4	EG	122	133,00	16226,00
	CG	122	112,00	13664,00
	Total	244		
criterii5	EG	122	136,00	16592,00
	CG	122	109,00	13298,00
	Total	244		
criterii6	EG	122	131,50	16043,00
	CG	122	113,50	13847,00
	Total	244		
criterii7	EG	122	133,00	16226,00
	CG	122	112,00	13664,00
	Total	244		

In Table 4, statistics from the Mann-Whitney test and significance levels for the seven criteria are presented. As can be seen, for all criteria, the significance is less than the chosen significance level of 0.01, which indicates that there is a statistically significant difference between the control and experimental groups at Stage 2 (the formative stage). This means that the experiment has achieved positive results.

Table 4. Statistics for the Mann-Whitney Test

	criterii1	criterii2	criterii3	criterii4	criterii5	criterii6	criterii7
Mann-Whitney U	5246,000	5490,000	6222,000	6161,000	5795,000	6344,000	6161,000
significance	,000	,000	,000	,001	,000	,004	,000

3.4. Quantitative and qualitative analysis of the results obtained before and after the pedagogical experiment for the experimental group

The conducted initial diagnosis aims to assess the changes in the mathematical preparation of 6- to 7-year-old children, as well as the effectiveness of the applied interactive learning model. The analysis of the results in this part of the study demonstrates the children's success in completing the assigned tasks compared to those of the control group, based on the initial criteria recorded after the experimental study.

To determine whether there is a statistically significant difference between the results before and after the experiment for the experimental group, the Wilcoxon signed-rank test for related samples was applied. The obtained results are presented in Table 5.

Table 5. Wilcoxon Test Statistics

Criterion	Wilcoxon Z Statistic	Test Significance
Criterii 1	–9,071	0,00
Criterii 2	–9,535	0,00
Criterii 3	–5,381	0,00
Criterii 4	–9,502	0,00
Criterii 5	–9,483	0,00
Criterii 6	–9,733	0,00
Criterii 7	–9,618	0,00

For all criteria, it was found that there is a significant statistical difference between the results before and after the experiment with the experimental group.

In conclusion, based on all the above, we can acknowledge the contribution of the technological model to the educational process of kindergarten children in mathematics, specifically in the core quantitative relations. The positive aspects of its application in the teaching and educational process are undoubtedly many, due to the fact that children acquire a rich set of knowledge and skills through games that are developmental in nature and have an interdisciplinary focus.

3.5. Results from a survey conducted with pedagogical specialists – SURVEY FORM – 2

After completing the pedagogical experiment aimed at proving or rejecting the main hypothesis H1 formulated in the dissertation, a second survey was conducted with the teaching staff who also participated in the first survey. The purpose of the survey was to explore the opinions of practitioners regarding the effectiveness of the presented technological model for pedagogical interaction to improve the intellectual and mathematical readiness of children in preparatory groups for first grade, which is indicated as an additional hypothesis H2 in this work.

Survey Form – 2 includes 5 questions, of which 60% (3 questions) have multiple-choice answers and 40% (2 questions) require open-ended responses. Each multiple-choice question offers different possible answers, with only one option to be selected. For the open-ended questions, the participants provide their personal opinions and give recommendations/evaluations on the discussed topic.

The analysis of the results from the experimental work and the surveys with kindergarten educators confirms the thesis that the model works successfully in a real pedagogical environment and leads to significantly better results in mathematics education in the preparatory group of the

kindergarten. This is indisputable evidence supporting the assumption that the implementation of the interactive learning model proposed in the dissertation will contribute to enhancing the intellectual and mathematical literacy of the children and prepare them for school.

CONCLUSIONS

The results of the conducted experimental work and the presented statistical analysis provide grounds to claim that the interactive technological model for teaching mathematics in kindergartens, proposed in the dissertation, yields positive outcomes in the educational process. Based on the in-depth analysis of the results from the experimental study, the following conclusions can be drawn:

1. The reviewed literature on the chosen topic made it possible to develop the concept of an interactive technological model for pedagogical interaction aimed at working with children in preparatory groups, within the studied core area of mathematics.

2. The survey conducted with educators – specialists in the field of preschool education – confirmed the need for developing an interactive technological model for working with 6- to 7-year-old children in the process of forming quantitative concepts.

3. The development and testing of an interactive technological model with a STEM orientation supports the formation of quantitative concepts in 6- to 7-year-old children, enhances their intellectual development, and activates the logical thinking of future first graders.

4. As a result of the conducted experimental work, it was found that in the experimental group there is a statistically significant difference between the results before and after the organized interactive pedagogical situations. In other words, the experimental group significantly improved their results after the intervention.

5. After the implementation of the experimental activities with the children in this group, a significant statistical difference was observed between them and their peers in the control group, with the experimental group achieving higher results in the final assessment. This indicates that the applied interactive technological method for pedagogical interaction in forming basic quantitative concepts in 6- to 7-year-old children yields positive results.

6. The proposed interactive model for working with children in the preparatory groups improved their mathematical literacy and increased their confidence in solving cognitive tasks.

CONCLUSION

Based on the conducted theoretical and empirical research regarding the formation of basic quantitative concepts in 6- to 7-year-old children, as well as the survey conducted with pedagogical specialists, the following hypotheses are confirmed:

1. The main hypothesis (H_1) of the study is confirmed, according to which, in the process of pedagogical interaction with 6- to 7-year-old children, the application of an interactive technological model improves the formation of basic concepts of quantitative relations in mathematics.

2. The second additional hypothesis (H_2) is also confirmed, stating that the implementation of an interactive approach increases the intellectual and mathematical readiness of children in preparatory groups for first grade.

3. The null hypothesis (H_0), which claims that there is no significant difference in the level of formation of basic quantitative concepts between children taught using the interactive model and those taught using traditional methods, is rejected.

SCIENTIFIC CONTRIBUTIONS OF THE ISSERTATION

*** Theoretical Contributions:**

1. An in-depth study of the theoretical and practical significance of the issue related to the formation of quantitative concepts in Bulgarian and international pedagogical practice for 6- to 7-year-old children.

2. Clarification of specific aspects of the educational and upbringing process of children in preparatory groups during the period of forming knowledge, skills, and competencies within the "Quantitative Relations" core area.

3. To ensure objectivity and comparability of the results, criteria and indicators for assessment were defined and introduced, in accordance with the needs of the presented dissertation.

4. Development of an original conceptual framework for interactive learning in early childhood education (ages 6–7), based on contemporary achievements in pedagogical and digital technologies.

5. Scientific justification of the relationship between the digital literacy of pedagogical staff and the effectiveness of forming quantitative concepts in preschool children.

6. Systematization of the interdisciplinary influences on early mathematical development through quantitative concepts, with a specific focus on preschool education.

*** Practical and Applied Contributions:**

1. A technological model of pedagogical interaction for the formation of quantitative concepts in 6- to 7-year-old children has been created and

tested. The model integrates interactive technologies, game-based methods, and interdisciplinary approaches.

2. A set of pedagogical situations and games with strong methodological focus has been developed to support the process of forming basic concepts and competencies related to quantitative relations in children from the preparatory group in kindergartens.

3. An interactive digital application has been developed, adapted to the age-specific characteristics of 6–7-year-old children, with options for both individual and group work.

4. Training sessions for educators were conducted to facilitate the adoption of the innovative model and its approaches, creating prerequisites for enhancing their professional competence and adaptability to contemporary educational challenges.

LIST OF PUBLICATIONS

1. **Angelova, K.** (2024). *Forming Basic Quantitative Concepts in 6–7-Year-Old Children through Role-Playing Games (Dramatization)*. *Yearbook of the Faculty of Education*. Paisii Hilendarski University Press, Plovdiv, Vol. 3, ISSN 2815-4134, pp. 331–344.

2. **Angelova, K.** (2024). *Forming Basic Quantitative Concepts in 6–7-Year-Old Children through the Application of a Technology for Solving Inventive Tasks*. In *Proceedings of the Interdisciplinary Scientific Conference of the Faculty of Education*. Paisii Hilendarski University Press, Plovdiv, ISBN 978-619-7768-12-1, pp. 439–450.

3. **Angelova, K.** (2024). *On the Role of Role-Playing Games (Dramatization) in the Formation of Basic Quantitative Concepts in 6–7-Year-Old Children*. In *Proceedings of the Sixth Scientific-Practical Conference "Current Policies and Practices in Education: Teaching and Learning for the Development of 21st Century Skills"*, organized by St. Cyril and St. Methodius University of Veliko Tarnovo, Pedagogical College – Pleven, ISBN 978-619-91255-9-5, pp. 321–328.

4. **Angelova, K**. (2023). *Theoretical Highlights on the Methodological Concept of Anna Leushina for Forming Basic Quantitative Concepts in 3–7-Year-Old Children*. In *Proceedings of the Twelfth Student Scientific Forum of the Faculty of Education*, Volume 1. Paisii Hilendarski University Press, Plovdiv, ISSN 2738-8859, pp. 27–33.

5. **Angelova, K.** (2023). *The Role of the Competency-Based Approach in Forming Basic Quantitative Concepts in 6–7-Year-Old Children*. *Scientific Journal of the Faculty of Education "Doctoral Research"*. Paisii Hilendarski University Press, Plovdiv, ISSN 2367-7309, pp. 48–57.