O P I N I O N

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Regarding a dissertation for the award of the educational and scientific degree "Doctor" in the field of higher education: 4. Natural Sciences, Mathematics, and Informatics, professional direction: 4.5. Mathematics, doctoral program: "Mathematical Analysis" at the Faculty of Mathematics and Informatics at Plovdiv University "Paisii Hilendarski" (PU).

Author of the dissertation: Laura Ajeti Azemi;

Dissertation topic: " On Coupled Fixed Points and Coupled Best Proximity Points for Cyclic, Noncyclic, and Semi-Cyclic Maps."

Scientific supervisors: Prof. DSc Boyan Zlatanov, Assoc. Prof. Dr. Hristina Kulina

1. General presentation of the procedure and the doctoral candidate

With Order No. RD-22-966 dated April 28, 2025, by the Rector of Plovdiv University "Paisii Hilendarski", I was appointed as a member of the scientific jury to participate in the procedure for the defense of a dissertation on the topic " On Coupled Fixed Points and Coupled Best Proximity

Points for Cyclic, Noncyclic, and Semi-Cyclic Maps ". This procedure is for obtaining the educational and scientific degree "Doctor" in the field of higher education: 4. Natural Sciences, Mathematics, and Informatics, professional direction: 4.5. Mathematics, doctoral program: Mathematical Analysis. The author of the dissertation is Laura Ajeti Azemi, who is a doctoral student at the Department of Mathematical Analysis under the supervision of Prof. DSc Boyan Zlatanov and Assoc. Prof. Dr. Hristina Kulina from Plovdiv University "Paisii Hilendarski". The submitted set of documents meets the requirements of the Law on the Development of Academic Staff in the Republic of Bulgaria, its implementing regulations, and the regulations for the development of academic staff at PU "Paisii Hilendarski".

2. Relevance of the Topic

I believe that the presented research is current and will attract the attention of a wide range of readers, including mathematicians and economists. Evidence of their significance is the high publication activity on the topic in prestigious international journals. The main results of the research in the dissertation have been published in peer-reviewed journals and presented at refereed conferences. Moreover, the first publication from 2022 already has four citations indexed in the Web of Science (WoS) and Scopus databases, which clearly demonstrates the relevance of the problems addressed in the dissertation.

3. Knowledge of the Problem

The topic of the research requires an in-depth study of the problem area. The doctoral candidate successfully demonstrates a high level of knowledge of the subject matter, which is evident from the well-structured dissertation. The introduction is divided into two parts: a preface describing the theme of the researched problems and Chapter 1, which serves as an introduction and presents the main concepts and theorems used in the following chapters. It is important to note that the bibliography includes only sources that are directly related to the research in the dissertation, without artificially added citations.

4. Methodology of the Research

The main tools for conducting the research in the dissertation are methods and scientific facts from several mathematical disciplines: real analysis, functional analysis, topology, variational principles, oligopoly market theory, and statistical methods for modeling based on data sets. Under the experienced guidance of her scientific supervisors, the doctoral candidate has mastered the specific methods and techniques from these areas and applied them correctly, ensuring good and reliable results.

5. Characteristics and Evaluation of the Dissertation and Contributions

Laura Ajeti Azemi's dissertation represents an in-depth innovative work related to generalizations of Banach's fundamental result on the existence and uniqueness of fixed points and consists of 136 pages. It comprises a preface, introduction, three chapters, conclusion, and a bibliography of 90 titles. In the dissertation, generalizations of Banach's fundamental result on the existence and uniqueness of fixed points are examined. These generalizations include coupled fixed points, tripled fixed points, coupled best proximity points, cyclic mappings, noncyclic mappings, and semi-cyclic ones. The underlying space is both uniformly convex Banach spaces and reflexive or metric spaces. The obtained results are illustrated with applications arising in the study of market equilibrium in oligopoly markets using real data. Additionally, a result generalizing Ekeland's variational principle over sets generated by mappings with mixed monotone property in partially ordered metric spaces has been derived. This result is then used to find sufficient conditions for the existence and uniqueness of tripled fixed points for ordered triples of mappings with the mixed monotone property. As a consequence, some already known results in the direction of tripled fixed points are obtained.

I fully accept all the scientific and applied contributions of the dissertation, evaluating them very highly.

I did not detect any plagiarism in the candidate's works according to the provisions of the Law on the Development of Academic Staff in the Republic of Bulgaria.

6. Assessment of Publications and Personal Contribution of the Doctoral Candidate

Laura Ajeti Azemi's publications related to her dissertation clearly demonstrate her significant contribution to the scientific community and confirm the quality of her work. The indexing of articles in prestigious databases such as Web of Science (WoS) and Scopus, as well as the fact that one of them is classified in the first quartile (Q1), testifies to the high level of scientific significance and recognition.

The four publications, including two in journals and two conference papers, cover a wide range of topics and methods, further emphasizing the diversity and depth of her research approach. The presence of four citations for a 2022 publication indexed in WoS and Scopus indicates that the results of her work are already finding their place in the scientific literature and are being recognized by other researchers.

The fact that Laura Ajeti Azemi exceeds the minimum national requirements according to the Regulations for the Implementation of the Law on the Development of Academic Staff in the Republic of Bulgaria by 105 points out of the required 30 is another proof of her productivity and competence. This, along with the successful dissemination of the results of the dissertation and her ability to work effectively in a team, makes her an excellent candidate for an academic career.

7. Abstract

The abstract of Laura Ajeti Azemi's dissertation is well-structured and accurately reflects the content of the main work. The volume of 64 pages is appropriate for providing a detailed overview of the research and its main results. Both the dissertation and the abstract comply with the requirements of the Regulations for the Conditions and Procedures for Acquiring Scientific Degrees and Holding Academic Positions at PU.

8. Recommendations for Future Use of Dissertation Contributions and Results

I do not have any criticisms regarding the provided documents and works. I recommend that Laura Ajeti Azemi continues working with the same diligence and expands the scope of her scientific research.

CONCLUSION

The dissertation includes results with scientific and applied significance, representing an original contribution to science. It fully meets the requirements of Bulgarian legislation and the internal rules of Plovdiv University "Paisii Hilendarski" for acquiring a scientific degree. Based on this, I give my positive assessment of the conducted research and recommend that the scientific jury awards the educational and scientific degree "Doctor" to Laura Ajeti Azemi in the field of higher education: "4. Natural Sciences, Mathematics, and Informatics," professional direction "4.5. Mathematics," doctoral program "Mathematical Analysis."

May 15, 2025 Prepared by: .....

/Assoc. Prof. Dr. Diana Kirilova Nedelcheva - Arnaudova/