OPINION

by Prof. Dr. Hristo Stefanov Kiskinov, PhD,

Professor at Plovdiv University "Paisii Hilendarski" (PU),

Faculty of Mathematics and Informatics

of a dissertation for the award of the educational and scientific degree **"Doctor" (PhD)**
by: Area of Higher Education 4. *Natural Sciences, Mathematics and Informatics*;
Professional Field 4.5. *Mathematics;*
Doctoral Program *Mathematical Analysis*.

Author: Laura Ajeti Azemi.

Title: "On Coupled Fixed Points and Coupled Best Proximity Points for Cyclic, Noncyclic and Semi-Cyclic Maps".

Scientific Supervisors: Prof. Dr.Sci. Boyan Georgiev Zlatanov and Assoc. Prof. Dr. Hristina Nikolova Kulina.

1. General Presentation of the Procedure and the PhD Student

By order No. PD-22-966 from 28.04.2025 of the Rector of the Plovdiv University "Paisii Hilendarski" (PU), I have been appointed as a member of the scientific jury to ensure a procedure for the defense of a dissertation (PhD-thesis) titled "*On Coupled Fixed Points and Coupled Best Proximity Points for Cyclic, Noncyclic and Semi-Cyclic Maps*" for awarding the educational and scientific degree "doctor" in the area of higher education *4. Natural sciences, mathematics and informatics*, professional field *4.5 Mathematics*, doctoral program *Mathematical Analysis*. The author of the dissertation is Laura Ajeti Azemi - PhD student in independent training at the Mathematical Analysis Department of the Faculty of Mathematics and Informatics (FMI) at Plovdiv University Paisii Hilendarski with scientific supervisors Prof. Dr.Sci. Boyan Georgiev Zlatanov and Assoc. Prof. Dr. Hristina Nikolova Kulina.

The set of paper and electronic materials presented by the PhD student Laura Ajeti Azemi is in full compliance with Article 36 (4) of the Regulations for the Development of the Academic Staff of the Plovdiv University and includes all the necessary documents.

Laura Ajeti Azemi is an Albanian by nationality and was born on 19.04.1993. In 2016, she obtained a bachelor's degree from the State University of Tetovo, in the city of Tetovo, Republic of North Macedonia, with a professional qualification of "Teacher of Mathematics", and in 2020, a master's degree again from the State University of Tetovo with a professional qualification of "Mathematician". From 2016 to 2023, she worked as a mathematics teacher, first at Skenderbeu High School in Preševo, Serbia, and subsequently at Ulpiana High School in Lipjan, Kosovo. Since 2023, she has been working as an assistant professor at the University UBT in Pristina, Kosovo. Since 2025, she has been a doctoral student in independent training at the Mathematical Analysis Department of the Faculty of Mathematics and Informatics at Plovdiv University Paisii Hilendarski.

2. Actuality of the Research Topic

Fixed point theorems have diverse applications in various fields of mathematics. Banach's famous theorem on contracting maps, published in 1922, is an extremely powerful tool in the theory of metric spaces. For more than a century, its various generalizations have been a fruitful subject of many mathematical studies. The various generalizations are based on several types of changes - of the space, of the map, of the contraction condition or of the notion of a fixed point, as well as on various combinations of them. The fact that the topic is particularly relevant today is easily established by following the publication activity of those working on this topic in specialized journals.

3. Knowing the Problem

The carried out review of the scientific research on the problem, the selected literature and above all the obtained results speak unequivocally of deep knowledge in the researched field.

4. Research Methodology

The methodology of the research is standard for most mathematical studies – proving statements, constructing methods and models based on the proven statements, their numerical implementation and presentation of illustrative examples.

5. Characterization and Evaluation of the Dissertation Work and Contributions - Presence/Absence of Plagiarism

The dissertation (PhD-thesis) "On Coupled Fixed Points and Coupled Best Proximity Points for Cyclic, Noncyclic and Semi-Cyclic Maps" is dedicated to various generalizations of the Banach's theorem for contracting maps, related to coupled fixed points, tripled fixed points and coupled best approximation points for cyclic, noncyclic and semi-cyclic mappings, as well as to their applications. It is written in English on 136 pages and consists of an introduction, four chapters, a conclusion and a bibliography with 90 titles. The presentation also includes 17 graphics and 16 tables. In the introduction, in addition to formulating the goals and tasks, some more significant results are highlighted, related to the possible generalizations of the Banach's theorem for contracting maps, which are directly related to the topic of the present dissertation. The first chapter has an overview character and it presents definitions and properties of the main concepts that are used later in the dissertation work. Chapters 2, 3 and 4 describe the conducted research and analyses, and the main scientific and applied scientific results obtained. Chapter 2 is devoted to coupled best proximity points for cyclic, semi-cyclic and noncyclic mappings in Banach spaces, Chapter 3 to variational principles in partially ordered metric spaces, and Chapter 4 presents various applications in modeling oligopolistic markets. In the conclusion, the doctoral student has made a self-assessment of the contributions contained in the dissertation, described the approbation of the obtained results and proposed specific perspectives for future research.

I support the main contributions described by the PhD student in the current dissertation. Namely:

- Coupled best proximity points for cyclic and semi-cyclic maps in reflexive Banach spaces instead of uniformly convex ones are considered.
- A technique for estimating the error for best proximity points for noncyclic maps has been developed.
- Coupled fixed points and tripled fixed points for maps with the mixed monotone property in partially ordered metric spaces are studied.
- Ekeland's variational principle for maps with the mixed monotone property is generalized and with its help conditions for the existence and conditions for the uniqueness of tripled fixed points for classes of such maps are found.
- Applications of some of the results in modeling oligopolistic markets are presented.

I do not detect "plagiarism" in the works of the author and the presented thesis in the sense of the Law on the Development of the Academic Staff in the Republic of Bulgaria.

6. Assessment of the Dissertation's Publications and Personal Contributions of the Author

The presented dissertation is based on 4 publications in English, two of which are in journals, one in conference proceedings and one conference report with a published abstract. Of the journal articles, one is in Mathematics, indexed in Web of Science with IF=2.258, Q1 and in SCOPUS, SJR=0.538, Q2, and the other in the journal AIP Conference Proceedings, indexed in Web of Science and in SCOPUS with SJR=0.152. The publications form a total of 105 points, which exceeds exactly three and a half times the minimum national criteria for this indicator, requiring 30 points.

Of the 4 presented articles, one is independent, two are with two authors and one with three. For me, the personal contribution of the candidate in the co-authored articles is undoubted.

It is important to be noted that so far 4 independent citations of one of the doctoral student's articles have been noted, all four in different reputable journals, each of which is indexed in Web of Science with IF and in SCOPUS with SJR.

I have no significant critical remarks.

7. Summary

The Summaries, written in Bulgarian and in English, have 64 pages instead of 32, as it should be according to the requirements of the Regulations for the Development of the Academic Staff of the Plovdiv University, and contains the main results obtained in the dissertation work.

8. Recommendations for Future Use of Dissertation Contributions and Results

I wish the PhD student to continue working in this interesting field with the same enthusiasm. The presence of concrete interesting opportunities for further development of the presented research, described in detail by the doctoral student, makes me confident that this will happen.

CONCLUSION

The dissertation ***contains scientific, scientific-applied and applied results, which are an original contribution to the science*** and **meet all the requirements** of the Law on the Development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for the Implementation of the LDASRB and the relevant Regulations of Plovdiv University "Paisii Hilendarski". I detect no plagiarism. The presented materials and dissertation results far exceed the minimum national requirements introduced by the Regulations for the Implementation of the LDASRB.

The dissertation work shows that the PhD student Laura Ajeti Azemi possesses in-depth theoretical knowledge and professional skills in the scientific specialty Mathematical Analysis, demonstrating qualities and skills for conducting research with obtaining original and significant scientific contributions.

Due to the above, I confidently give my **positive assessment** of the conducted research, presented by the above-reviewed PhD thesis, summary, achieved results and contributions, and ***I propose to the honorable scientific jury to award the educational and scientific degree "Doctor" (PhD)*** to Laura Ajeti Azemi in the Area of Higher Education 4. *Natural sciences, Mathematics and Informatics*; Professional Field 4.5. *Mathematics;* Doctoral Program *Mathematical Analysis*.

13.05.2025 Scientific jury member: .....

Plovdiv Prof. Dr. Hristo Stefanov Kiskinov