R E V I E W

by DSc Angel Borisov Dishliev,
professor at the University of Chemical Technology and Metallurgy – Sofia
on:

Dissertation work for awarding the educational and scientific degree "doctor";
Field of higher education: 4. Natural sciences, mathematics and informatics;
Professional direction: 4.5. Mathematics;
PhD program: Mathematical Analysis;
Author of the dissertation: Mira Lachezarova Spasova;
Thesis topic: Analytical methods for solving some classes of fuzzy integro-differential equations;
Research supervisor: Prof. Dr. Atanaska Tencheva Georgieva from Plovdiv University "Paisii Hilendarski"

1. General description of the presented materials

By order No. RD-21-454 of 23.02. 2024 of the Rector of Plovdiv University "Paisii Hilendarski" (PU) I was appointed as a member of the scientific jury to ensure a procedure for the defense of the above-mentioned dissertation work. At the first meeting of the jury, I was selected as a reviewer.

The set of materials presented by the doctoral student Mira Spasova on an electronic medium is in accordance with Art. 36 (1) of the Regulations for the Development of the Academic Staff of the PU. The main document is of course the dissertation.

The dissertation is placed on 107 standard pages. Consists of:
- introduction;
- four chapters (each containing several paragraphs);
- a conclusion that represents:

  - author reference of the contributions in the peer-reviewed work. A total of seven main contributions are indicated;
  - list of publications on the dissertation - a total of 4 publications, all co-authored with the supervisor;
  - declaration of originality of the results;
  - citations – one citation by foreign authors in a journal with IF;
  - approval of the results - a total of five conference reports, one of which is abroad;
  - participation in projects – a total of three projects of university importance;
  - declaration of originality;
  - bibliography including 103 titles.

In addition to the dissertation, the doctoral student has attached to the documents for the procedure for obtaining the educational and scientific degree "doctor":
- Abstract;
- Declaration of originality and authenticity;
- Resume (European format);

- Copies of four scientific articles, published in full text and which are related to the topic of the dissertation work. The authors of these publications are M. Spasova and her scientific supervisor. It may be added that the dissertation is based on these publications;
- Other documents that are related to the procedure for the defense of the dissertation work and are required by the relevant regulations. In the review, I will not comment on these documents, as the aim is to concentrate only on the essential documents and achievements of the candidate. Although "it's not my job", I will still note with satisfaction that the amount of documents a PhD student has to provide before the defense is significantly less than a few years ago. This is undoubtedly good for these young people, for nature, and at the same time it is bad for the "paper industry".

2. Brief biographical data for the doctoral student

Young people have relatively short biographies, and they have not yet learned to "stretch locums", describing every breath in their autobiographies. The candidate for the acquisition of the educational and scientific degree "doctor" completes successive educational qualification degrees (EKD) specified in the following table:

Period from - to	EKD	Qualification	School
2009-2014	Average		Secondary School "Bertolt Brecht", Pazardzhik town
2014-2018	Bachelor	Teacher of mathematics, informatics and information technologies	PU "Paisii Hilendarski"
2018-2019	Master's degree	Mathematician	PU "Paisii Hilendarski"
2020-2023	Doctor (full-time education)	Doctor - Doctoral Program: Mathematical Analysis	PU "Paisii Hilendarski"
01.03.2023	Charged with right of defence		PU "Paisii Hilendarski"

Table 1

Mira's employment so far is as follows:

Period from - to	Occupation	Workplace
2016-2017	Teacher of informatics and information technologies	SS "Chernorizets Hraber", City Plovdiv
2018- until now	Teacher of mathematics and information technology	Sports School Pazardzhik town

Table 2

3. Actuality of the topic and appropriateness of the set goals and tasks

Famous scientists (specialists in applied mathematics and informatics) define the theory of fuzzy sets and related applications as the "third wave of intellectual programming". The theory was created by the American scientist Lotfi Zadeh in several seminal studies starting more than half a century ago (the first work was in 1965). The aforementioned American scientist lays out the foundations of the theory, introduces the basic concepts and, most importantly, proves the need to use fuzzy sets and the logic associated with them as a fundamental mathematical apparatus for modeling imprecise, indeterminate, qualitatively set statements and inferences (and not absolutely precisely, i.e. quantitatively). The scientific community did not take the new theory very kindly. The majority of scientists (at that time) accepted the theory with certain "reservations". For example, in the 1990s, the theory and applications of this modern science were removed (albeit briefly) from the curricula of several elite American universities. On the other hand, in parallel with these "bans", technologies are recommended and advertised that use methods that are based on fuzzy set theory. Conceptually, fuzzy mathematics can be seen on the one hand as a generalization of classical mathematics, and on the other - as an opposition to "exact" mathematics. In this way, a more adequate form for modeling the natural phenomena and real processes caused primarily by human intervention is introduced and exploited. This is achieved by the fundamental assumption in this theory that the notion of belonging of a certain element to a given set (which in this theory is called a "fuzzy set") can change "smoothly" from "unequivocally belongs" to "unequivocally does not belong". From here also follow the statements specific to fuzzy mathematics, which have a variable (generally continuous) degree of truth: from absolutely true to absolutely false. This modern scientific direction is developing extremely rapidly, which is expressed in numerous scientific articles (in renowned scientific journals), monographs, scientific conferences, congresses, workshops, etc. Here I will recall the colloquiums on Numerical methods and applications with the chairman of the organizational board prof D. Bainov, held annually at TU - branch city of Plovdiv. These colloquiums were organized for more than ten years - at the end of the last century and the beginning of the present century. In the thematic direction of the work of the mentioned colloquiums, one of the scientific directions was declaratively the theory and applications of fuzzy sets. At present, the theoretical foundations of the theory of fuzzy mathematics have been established and wide applications in practice are forthcoming. In 1975, E. Mamdani, together with a group of students, created the first practical application of the new theory - a fuzzy controller for controlling a steam boiler. In the early 1990s, industrial giants Motorola, General Electric, Otis Elevator, Pacific Gas & Electric, Ford, IBM, and others invest in the development of products that use fuzzy mathematics in their automated activity. As a result of large investments, numerous technical and industrial developments based on fuzzy control mechanisms are created. Examples include: electric locomotives, helicopters, cars, video cameras, air conditioners, vacuum cleaners, washing machines, freezers, etc. We can definitely say that today fuzzy set theory and the resulting fuzzy logic are widely used as a means of formalizing uncertainties in many software packages. Their purpose is to analyze crisis situations, financial markets, political situation, etc. A prerequisite for their adequacy is the modern complexity of the processes resulting from the situations of uncertainty and unpredictability.

At the end of this point, I will summarize my belief that the results in the thesis are up-to-date and in sync with current trends in the theory of fuzzy integral equations. The place of the research presented in the dissertation will, in due course, be determined by the interest it will arouse in the mathematical community.

4. Knowing the problem

I consider that the PhD student knows:

- the mathematical objects of research in the scientific work submitted for review;
- the historical development of the considered scientific problems;

- the current state of this relatively new mathematical direction;
- some directions in which the theory of fuzzy mathematics is to be developed;
- specific opportunities for application of the achieved theoretical results;

The most important thing, in my opinion, is that Mira Spasova is able to carry out independent scientific research.

I come to these conclusions bearing in mind:

- The author's serious, content-rich and foundational introduction and introduction (Chapter 1) to the topic of the dissertation. Basic definitions and results of leading domestic and foreign authors, on which the studies are based, are indicated. Sufficiently specific applications of the considered types of integral equations are presented, which once again convince us of the reliability of the presented results;
- When reading, even during the initial acquaintance with the scientific work, it is not necessary to use additional, introductory, reference literature on the subject. In other words, the beginning of the dissertation has the quality of a teaching aid (I will specify - for advanced students).
- On the one hand, the above circumstance is convenient for the professional reader without prior preparation on the topic of the dissertation, and on the other hand, it confirms the opinion expressed above about the detailed knowledge of the genesis of fuzzy sets, numbers, functions, derivatives, integrals, some classes of equations and relevant properties and operations on these objects;
- The fluent command of the terminology and the basic definitions of the topic and the ability to combine defining qualities of different mathematical objects is the basis for my statement about the doctoral student's competence in the issues discussed in the dissertation;
- The deep insight into the essence of fuzzy sets and the alternative fuzzy mathematics originating from them (operating with concepts that carry the properties of indeterminacy, inaccuracy and the presence of elements of qualitative determination of the objects, phenomena and processes described by these concepts) and the essential difference with the classical mathematics convinces me of the good preliminary scientific preparation of the doctoral student on the topic of the dissertation. The research supervisor probably also has merit here;
- The indicated literature used and some comments on the works of other authors represent a confirmation of the author's involvement with the considered class of integral equations;
- The multitude of essential remarks and consequences (some of them of independent interest), which clarify and complement the theoretical results of the author of the dissertation work, give me reason to consider that the theory of these complex mathematical objects is deeply thought out;
- In several places in the reviewed work, it can be seen that the doctoral student is able to creatively apply known results and research methods of other authors. It has the quality to reasonably supplement and adjust the limitations imposed on the objects under consideration, and in some cases to overcome serious difficulties of a technical nature.

I also answer the recently standard question regarding the originality of the results:

The dissertation lacks elements of repetition and plagiarism by foreign researchers.

I conclude with a summary of the presentation made on the point: Mira is thoroughly familiar with the scientific issues discussed in the dissertation and possesses the necessary knowledge and skills to conduct independent research.

5. Research methodology

The main apparatus of conducting research in the peer-reviewed work are methods and some scientific facts from several mathematical sciences:

- real mathematical analysis (the ubiquitous use of analysis when formulating all definitions and statements, as well as when performing the relevant proofs in the peer-reviewed work);
- functional analysis (basic ideas from this science are used in the formulations and proofs in the dissertation - for example, the Adomian decomposition method and the Sumudu decomposition method are applicable to a wide variety of functional equations. In addition, part of the research is carried out in abstract spaces (e.g. a Banach space). Therefore, we can consider that the technique of functional analysis is used in the thesis);
- interval analysis (in its various varieties, it is used as the basic apparatus of arithmetic operations between fuzzy sets and fuzzy numbers, presentation of the given and sought fuzzy functions and the operations between them, finding appropriate derivatives of this type of functions and their integrals);
- fundamental theory of some types of integral equations (theorems for existence and uniqueness of solutions of classes of such equations are basic reference information for research in the dissertation);
- qualitative theory of integral equations (use of different properties of the solutions of the studied classes of integral equations is information that is intended to be adapted for the corresponding fuzzy equations).

As in all mathematical dissertations (with which I have hitherto been acquainted), so here, no single (isolated) mathematical method is used, and only that method which is applied to suitable objects in order to discover new facts. The use of different knowledge, methods, ideas from several mathematical sciences, etc., their appropriate creative combination, in order to achieve new results - this is the scheme by which the research in the peer-reviewed dissertation work is carried out. I will note that this way of uncovering new theoretical and applied facts is difficult and is inherent to researchers with developed creative potential.

I think that this approach to identifying the goals and tasks to solve, the methods of research work, is transmitted to the doctoral student by his supervisor.

6. Characterization and evaluation of the dissertation work

The Introduction of the dissertation is informative. It starts by pointing out the need to study the numerous types of equations containing the sought-after function under the sign of the integral (which in the following we will generally call integral equations). In particular, this broad class of equations includes the fuzzy integro-differential equations, which are the main object of research in the dissertation. The main reason for using the fuzzy integral equations as a modeling mathematical apparatus is also clearly formulated - namely: the "inaccuracy" (or "uncertainty", "incompleteness", etc.) of the output initial data and the corresponding defining functions. A brief (rather introductory) historical review of the development of the fundamental and qualitative theory of fuzzy integral equations is made. Some basic methods for solving them are indicated.

The second part of the introduction contains the goals that the author sets for himself and the way to achieve them. In other words, the consecutive specific mathematical tasks are marked, through which the set goals are realized. Briefly, the objectives are as follows:

- To adapt and create a mathematical apparatus necessary to guarantee the existence and uniqueness of the solutions of initial problems for certain classes of linear and non-linear fuzzy integro-differential equations;
- To prepare a suitable technique and to prove preliminary properties of the mathematical apparatus, which are necessary for solving initial and boundary problems for fuzzy partial integro-differential equations;
- To construct decomposition methods for finding approximate solutions of a fuzzy Volterra-Fredholm integro-differential equation;
- To adapt analytical methods that use the Sumudu and Natural fuzzy transforms to find solutions of Volterra's linear fuzzy integro-differential equation and Volterra's linear partial fuzzy integro-differential equation.

The third part of the introduction is an overview of the dissertation work. The content of the individual chapters and their adjacent paragraphs is briefly synthesized.

The First chapter of the dissertation has an introductory-informational character. First, the basic definitions of fuzzy sets and operations between them are given. Further, fuzzy numbers are defined using fuzzy sets (as a special class of them). Various types of fuzzy numbers are presented, such as: triangular, trapezoidal, Gaussian, etc., as well as their geometric interpretations. The introduction of arithmetic operations between fuzzy numbers is based on interval analysis and the so-called "levels" of fuzzy numbers (which levels are essentially intervals with variable boundaries, depending on the desired level). Basic properties of arithmetic operations between fuzzy numbers are indicated, and what is specific and useful: differences with arithmetic in the classical real case are highlighted. The concept of distance between two fuzzy numbers is defined by the Hausdorff metric. The "subtraction" operation in interval analysis has a layering nature. In one of the directions of development of this science, the subtraction operation is perceived in a set-theoretic sense, in another direction it is in the sense of Hukuhara, and in a third - in the generalized sense of Hukuhara, etc. This circumstance is discussed in the dissertation, being noted the advantages and disadvantages of all these "differences" and their relationship.

Fuzzy functions (or as Prof. B. Sendov called them "thick" functions) are described by fuzzy numbers. The definitions of bounded and continuous fuzzy functions are given. In the introductory chapter, the main place is given to the description of different types of derivatives of fuzzy functions. Hukuhara's generalized difference is used in the relevant definitions. A relationship between the derivative of a fuzzy function and the derivatives of the corresponding lower and upper level functions is indicated. The definition of fuzzy Henstock integral is given. As a special case, the Riemann integral is also obtained, in which the δ grid of breaking up the integration interval is defined by a constant function, i.e. $\delta =$

$const$. The main properties of the Henstock integral are given, which are similar to the properties of the Riemann integral of a classical (numerical) function of one argument. Several "working" properties of the nonproprietary integral of a fuzzy function in the Riemann sense are indicated. Similar questions are also addressed for fuzzy functions of two arguments.

Finally, the chapter examines a fuzzy analogue of the Volterra model for population growth within a given closed system. This model is described by the fuzzy nonlinear Volterra integro-differential equation, which we will discuss next

I will not fail to express my pride in the fact that a number of Bulgarian scientists, as well as the schools founded by them, received fundamental statements in several "emerging" mathematical theories at that time, such as: interval analysis, the theory of the Hausdorff metric and the theory of fuzzy sets. Several monographs have been published on these topics, which have generated serious interest. I will add that the research of Bulgarian mathematicians started already in the seventies of the last century, i.e. at the beginning of the construction of the mentioned mathematical theories. Here I will announce the names of B. Sendov and S. Markov, with whom I had the opportunity to work on these topics.

The Second chapter is basic in the dissertation, as it presents the ideas of the basic working approach for solving initial problems for classes of fuzzy integro-differential equations. Specifically, a complex and marginally general initial problem for a fuzzy integro-differential equation of the form is studied:

$$\sum_{j=0,\dots,k} p_j \odot u^{(j)}(x) = g(x) \oplus (FR)\int_a^x K_1(x,s) \odot G_1\big(u(s)\big)ds \oplus (FR)\int_a^b K_2(x,s) \odot G_2\big(u(s)\big)ds, \qquad \text{(A)}$$

$$u^{(j)}(a) = b_j, \quad j = 0,1,\dots,k-1,$$

where: the symbol (FR) means that the integral following it is in the Riemann sense; the characters $\odot$ and $\oplus$ are for the multiplication and addition operations in fuzzy arithmetic, E^1 is the set of fuzzy numbers; $k \in N$ is a natural number indicating the order of the highest derivative in the considered equation, the numerical constants $a,\ b \in R$; the fuzzy constants $b_0,\dots,b_{k-1} \in E^1$, the numerical functions $p_j: [a,b] \to R$ and $K_1, K_2: [a,b] \times [a,b] \to R$ are continuous in the definitions are multitudes; the fuzzy functions $g, u: [a,b] \to E^1$ are also continuous on the interval $[a,b]$; the operators $G_1, G_2: E^1 \to E^1$are continuous in E^1. The main stages in the research in which the solution method is constructed are preceded by:

- Presentation of a list of easily verifiable natural conditions on the elements of the studied equation and the corresponding initial conditions. These requirements are imposed in order to guarantee the existence of the studied mathematical objects;
- Fuzzy numbers and functions involved in writing equation (A) are presented in parametric form;
- Output versions of equations are obtained, in which the lower and upper functions at the level of the unknown fuzzy function in (A) participate, respectively;

- The original equations are transformed by multiple integrations (as many times as the order of the highest derivative in the studied equation), using the initial conditions and known formulas and equalities from the analysis.

In this way, a clear parametric form of the output equation is obtained. More precisely, two (traditional, i.e. not fuzzy) integro-differential equations are obtained, which are satisfied by the lower and upper functions of the parametric representation of the sought function, respectively.

From here on, the main research on the topic starts. To the mentioned two equations (which we will emphasize once again that they are not fuzzy), the author of the dissertation applies the Adomian decomposition method to find their solutions exactly or approximately. The method consists in a formal representation of the unknown functions by an infinite functional series. In this case, the unknown functions are considered to be the lower $\underline{u}(x,r)$ and upper $\overline{u}(x,r)$ functions at the level of the sought function $u(x)$. Formal lines have the usual form:

$$\underline{u}(x,r) = \sum_{i=1,2,\dots} \underline{u_i}\,(x,r), \quad \overline{u}(x,r) = \sum_{i=1,2,\dots} \overline{u_i}(x,r). \tag{B}$$

The terms of the formal series (B), which refer to the functions of the lower level $\underline{u}(x,r)$ and the upper level $\overline{u}(x,r)$, are determined successively by recurrence formulas. The recurrence formulas are obtained after substituting the lines in the equations for $\underline{u}(x,r)$ and $\overline{u}(x,r)$. All these transformations are possible with additional requirements on the qualities of the involved functions and constants in the input of the original equation (A).

An important part of the author's reasoning concerns the specification of conditions guaranteeing the existence and uniqueness of the solution to equation (A). Again, this question boils down to the existence and uniqueness of the lower and upper functions at level (B).

It is clear that in practice the solution of equation (A) cannot be found exactly by the applied Adomian method. As we said above, the lower and upper level functions are found by solving countably many repeated equations whose solutions fill the rows specified in (B). The conclusion made naturally leads to the conclusion of an approximate approximation of the functions $\underline{u}(x,r)$ and $\overline{u}(x,r)$ by their partial sums:

$$\underline{s_n}(x,r) = \sum_{i=1,2,\dots n} \underline{u_i}\,(x,r), \quad \overline{s_n}(x,r) = \sum_{i=1,2,\dots n} \overline{u_i}(x,r). \tag{C}$$

Sufficient conditions were found from which the convergence of the series (B) follows. In a suitable Banach space, an estimate of the error between the exact and approximate solutions is obtained, i.e. are assessed:

$$\left\|\underline{u}(x,r) - \underline{s_n}(x,r)\right\| \text{ and } \|\overline{u}(x,r) - \overline{s_n}(x,r)\|.$$

The second (main) topic discussed in the second chapter of the dissertation is devoted to Sumudu's fuzzy decomposition method. I briefly digress and state that I can see no good reason why the research on this (second) topic should not be presented in a separate chapter. In order to present the method in a fully justified way, suitable introductory preconceptions are given and necessary statements concerning the fuzzy Sumudu transform The transform maps the original $w(x)$ to the image $W(u)$ and has the form:

$$W(u) = W_S(u) = S[w(x)] = (FR)\int_0^\infty e^{-x} w(ux)dx, \tag{D}$$

where $w: R^+ \to E^1$ is a continuous fuzzy function and the integral in (D) exists. Formula (D) is an analogue of the classical Sumudu transformation. Sufficient conditions for correctness of the transformation are presented. The main assumption is that the original fuzzy function $w = w(x)$ is of a suitably chosen exponential order α, i.e.:

$$(\exists M = const > 0)(\exists \alpha = const > 0)(\exists X = const > 0):\ \ (\forall x > X) \Rightarrow D(w(x), 0) \le Me^{\alpha x}, \tag{E}$$

where the symbol D denotes the Hausdorff distance. A closer look at the transform (D) shows that it is "close" to the Laplace transform:

$$W(u) = W_L(u) = L[w(x)] = (FR)\int_0^\infty e^{-sx} w(x)dx. \tag{F}$$

This circumstance gives reason to assume that the properties of these two transformations are also "close". Several properties of the fuzzy Sumudu transform are proved, such as: linearity of the transform; image of exponential decay or growth of the original; image of periodical original and others. The results are similar to the classical results known for numerical functions and the corresponding classical Sumudu transforms. Moreover, the properties resemble those of the Laplace operator. The proofs, as in previous studies, are carried out piecewise - sequentially with the lower and upper level functions. A special section is devoted to the study of the fuzzy convolution of two functions, which resembles the classical convolution, under both the Sumudu operator and the Laplace operator. We have:

$$(v * w)(x) = (FR)\int_0^x v(x-s)w(s)ds,$$

where the fuzzy functions $v, w: R^+ \to E^1$. It is confirmed that (as with numerical functions) the image by the Sumudu transform of the convolution of two fuzzy functions is the product of their images. To the preparatory results in this second part of the second chapter, it is shown that the derivative of fixed order n of the Sumudu transform of a sufficiently differentiable fuzzy function is equal to the transform of the derivative of order n of the given fuzzy function. Of course, one of the basic requirements here is that the transformation exists. In the main part of this second topic, the fundamental theory of the following initial problem for a fuzzy Volterra-Fredholm integro-differential equation is constructed:

$$w^{(n)}(x) = g(x) \oplus (FR)\int_0^x K_1(x-s) \odot G_1\big(w(s)\big)ds \oplus (FR)\int_0^b K_2(x-s) \odot G_2\big(u(s)\big)ds;$$
$$w^{(i)}(0) = b_i, \quad i = 0,1,\dots,n-1, \tag{G}$$

where: $n \in N$ is a natural number; the numerical functions $K_1, K_2: [0,b] \to R$; the fuzzy function $g: [0,b] \to E^1$; operators $G_1, G_2: E^1 \to E^1$ are continuous in E^1; the fuzzy constants $b_0, \dots, b_{n-1} \in E^1$, the constant $b \in R^+$. The developed algorithm for finding the solution of the fuzzy equation (G) can be broken down into the following stages, observing their implementation sequence:

- Sumudu fuzzy transform is applied to both sides of the equation;
- Transformations are performed using transformation properties related to images of derivatives and convolutions of fuzzy functions;
- Through the parametric form of the fuzzy numbers and fuzzy functions involved in the considered equation, as well as the initial conditions, equation (E) is reduced to a system of two traditional (numerical) integro-differential equations of the Volterra-Fredholm type;
- For the latter system, the Adomian decomposition method for traditional equations is applied. As we said before, the idea of the method is to represent the unknown functions formally in infinite function series;
- The elements of the rows are found by enumerably many recursive formulas;
- Finally, the inverse Sumudu transformation was applied and the lower and upper solutions of the original equation (E) were obtained;
- From the last fuzzy solution of the discussed equation (which in principle is unattainable - due to the enumerable number of recurrent formulas) it is possible to find an approximate solution, as a partial sum of the lines in question.

In **Chapter Three**, Natural's fuzzy transform is constructed and applied to find the exact solution of a relatively narrower class of fuzzy equations, more precisely: Volterra's first-order fuzzy linear integro-differential equation with a convolutional kernel. The derivative of the unknown function is of the Hukuhara type. Specifically, the following initial problem is studied:

$$\int_0^x K_1(x-s) \odot w(s)ds \oplus \int_0^x K_2(x-s) \odot w^{(n)}(s)ds = g(x) \tag{H}$$

$$w^{(i)}(0) = b_i, \quad i = 0,1,\dots,n-1,$$

where: $n \in N$; the functions $K_1, K_2: [0,b] \to R$; the fuzzy function $g: [0,b] \to E^1$; the fuzzy constants $b_0, \dots, b_{n-1} \in E^1$, the constant $b \in R^+$.

The Natural operator maps the fuzzy original function w(x) into the fuzzy image - the function $W(s,u)$. We have:

$$W(s,u) = W_N(s,u) = N[w(x)] = (FR)\int_0^\infty e^{-sx} w(ux)dx, \tag{I}$$

where $w: R^+ \to E^1$ is a continuous fuzzy function and the integral in (I) exists. Clearly, the Laplace and Sumudu transforms are particular cases of (I). They are obtained for $u = 1$ and $s = 1$, respectively. The most important properties of the Natural operator, which are necessary in constructing the algorithm for solving the equation (H), are given:

1. The relation of the Natural operator $W_N(s,u)$ to the Laplace operators $W_L(s)$ (see (F)) and Sumudu $W_S(u)$ (see (D)) is given. For natural constraints, the equalities are valid:

$$W_N(s,u) = \frac{1}{u} W_L\left(\frac{s}{u}\right) = \frac{1}{s} W_S\left(\frac{u}{s}\right);$$

2. The inverse transformation is indicated, and the form of the level functions is also given;

3. Sufficient conditions for the existence of the transformation of Natural are presented. The most important condition is that the original is of exponential order (see (E));
4. Several images of the operator for originals are given, which are common elementary functions;
5. Basic properties of the operator are formulated and proved. For example, under certain conditions the equalities are valid:

$$N[c_1 \odot w_1(x) \oplus c_2 \odot w_2(x)] = c_1 \odot N[w_1(x)] \oplus c_2 \odot N[w_2(x)] \ ;$$

$$N[w(ax)] = \frac{1}{a} W\left(\frac{s}{a}, u\right); \qquad N[e^{-ax} \odot w(ax)] = W(s+a,u) \ \text{ and others;}$$

6. The image of the convolution of two fuzzy continuous functions is presented;
7. The image of the derivative of an arbitrary order of a fuzzy function is indicated. The derivative is in the Hukuhara sense. With natural constraints, an expected result was obtained, namely:

$$N\left[\frac{d^m w(x)}{dx^m}\right] = \frac{d^m}{dx^m} \ N[w(x)];$$

8. Useful relations are found between the image of a fuzzy function and the images of its lower and upper level functions.

After the preliminary remarks, a brief content of which can be seen above, the author has proposed an algorithm for finding the solution of the problem (H). The main steps in the algorithm are as follows:
1. The Natural operator is applied to both sides of the equation indicated in (H);
2. The newly obtained equation is transformed using the properties of the discussed transformation;
3. Algebraic equations for the lower and upper level functions are found;
4. The solutions of the original problem are obtained by the inverse Natural transformation.

The **Fourth chapter** is devoted to the development of an algorithm for finding the solution of a fuzzy Volterra partial integro-differential equation of a special class. The method presented in the thesis includes the fuzzy Sumudu transform for a function of two variables and the fuzzy two-dimensional Natural transform. Consider the equation:

$$\sum_{i=1,\dots,m} a_i \odot \frac{\partial^i w(x,t)}{\partial x^i} \oplus \sum_{j=1,\dots,n} b_j \odot \frac{\partial^j w(x,t)}{\partial t^j} \oplus c \odot w(x,t) \qquad \text{(J)}$$

$$= g(x,t) \oplus (FR) \int_0^t K(t-s) \odot w(x,s) ds$$

with initial and boundary conditions of the form:

$$\frac{\partial^j w(x,0)}{\partial t^j} = \psi_j(x), \quad 0 \le x \le b, \quad j = 0,1,\dots,n-1; \qquad \text{(K)}$$

$$\frac{\partial^i w(0,t)}{\partial x^i} = \varphi_i(t), \quad 0 \le t \le d, \quad i = 0,1,\dots,m-1,$$

where: $m, n \in N$ are natural numbers; the constants $a_1, \dots, a_m,\ b_1, \dots, b_n,\ c \in R$; the constants $b, d \in R^+$; the number function $K: [0, d] \to R$ is continuous; the fuzzy function $g: [0, b] \times [0, d] \to E^1$ is continuous; the fuzzy functions $\psi_j: [0, b] \to E^1$ and $\varphi_i: [0, d] \to E^1$ are continuous. The developed method is applied to the problem (J), (K). Conceptually, the studies here do not differ significantly from those in the previous chapters, so we will not consider them.

7. Contributions and significance of the development for science and practice

The contributions are formulated correctly by the doctoral student and are placed in several places in the defense documents - for example, in the Conclusion to the dissertation, also in the Author's abstract, etc. The achievements in the dissertation can be attributed to the thematic enrichment of the fundamental theory, qualitative theory and approximate methods for finding the solutions of special classes of fuzzy ordinary and partial integro-differential equations. I agree with all the contributions declared by the candidate for the acquisition of the educational and scientific degree. Although I will repeat part of the previous point 6 of the review, I will briefly describe the contributions that have been made in the first part of the second chapter. In my opinion, these contributions are essential and also provide insight into the achievements in the next two chapters of the thesis:

- The formulation of the considered initial problem for a fuzzy integro-differential equation of a wide class is given;
- The solution of the fuzzy equation is reduced to solving a pair of initial problems for traditional (non-fuzzy) integro-differential equations. The unknown functions in the traditional equations are a lower and upper level function for the fuzzy solution of the source equation;
- The lower and upper level functions are represented formally as function lines;
- Adomian's method was applied to find the members of the mentioned functional lines;
- Convergence of the discussed lines is proven;
- An error estimate was found when replacing the lower and upper level functions with partial sums of their respective rows.

In my opinion, the research in the dissertation is sufficiently deep and important that it will occupy a worthy and lasting place in science.

8. Evaluation of publications on the dissertation work

The dissertation work is based on 4 scientific publications with the participation of the doctoral student. Each of these publications is by two authors (the PhD student and his supervisor). All scientific works have been published in AIP Conference Proceedings The relevant reports have been presented at the conferences:

- *International Conference Applications of Mathematics in Engineering and Economics* (two reports);
- *International Conference New Trends in the Applications of Differential Equations in Sciences* (two reports).

Two of the reports were conducted in 2021, when the journal had SJR=0.189, and the other two reports were in 2022, when the journal had SJR=0.164.

As can be seen, the publications are relatively recent, which is why it is too early to expect citations of the obtained scientific achievements. So far, one citation by foreign authors has been noticed, which was placed in a reputable journal.

I will explicitly emphasize that the publication activity of the doctoral student related to his doctoral studies is more than satisfactory. The publications are not many in number, but they are published in a prestigious scientific publication, which is why they are thoroughly covered by various secondary databases. Therefore, peer-reviewed scientific achievements are easily followed by the interested scientific community. This circumstance, in my opinion, will lead to the use of the results and their citation in the near future.

Part of the results in the dissertation work were used in the work on three scientific projects. Two of the mentioned projects are at the university level and one is at the national level.

Dissertation results have been reported at 5 scientific forums. Four of them are at the conferences mentioned above in this point and one at the conference:

Second International E-Conference on Mathematical and Statistical Science: A Selcuk Meeting (ICOMSS'23), Selcuk, Turkey.

In conclusion of this point of the review, I will note that the scientific publications with which the doctoral student participates in the defense of the dissertation exceed the minimum national requirements. For clarity, the following table presents the minimum national requirements and the corresponding indicators achieved by the doctoral student:

A group of indicators	Minimum number of points required by law	Submitted materials by the applicant	Number of points of the candidate
A. Dissertation work	50	Dissertation: Analytical methods for solving some classes of fuzzy integro-differential equations	50
G. Scientific publications	30	Publications: - 4 publications in SJR-classified journals: ⇒ 4x30=120 points.	120

Table 3

9. Personal participation of the doctoral student

Each scientific paper is based on the work of several scientists (who may not be connected in a working group and may not be the authors of the presented research). Here I will locate my "philosophical reflections" and indicate the most important researchers involved in the work on a given dissertation:

- **The scientific supervisor** who, according to accepted practice (although not necessarily), is the author of:

1. the selection of the used and studied mathematical objects - it is also possible to introduce new ones;
2. the topic of the dissertation;
3. selection of appropriate scientific literature on the topic of the dissertation
4. the statements of part of the tasks, which represent the content of the dissertation work and must be solved consistently in the relevant studies;
5. methods for solving the tasks, including: baseline research, possible options for solutions, etc.;

6. expected results and determination of their place in the scientific literature;
7. possible applications of the achieved results, etc.;

- **The doctoral student** who is responsible for the entire dissertation must:

1. thoroughly and purposefully get acquainted with the source information on the topic of the dissertation;
2. to technically prepare for writing and subsequent layout of the dissertation work;
3. on the basis of the pre-determined main tasks, reach a concrete formulation of the theorems and statements originating from them;
4. to participate directly in the proofs of the statements, while his participation is decisive in the scientific work in some (and why not in most) parts of the dissertation;
5. to carry out a "research content arrangement", which consists in evaluating the results that are included in the dissertation and discarding some previously achieved results that are extraneous and so to speak blur the "theory of fuzzy equations" or on the other hand, they break the logical thread of the narrative in the work;
6. to carry out a mandatory secondary review of the restrictions (conditions) guaranteeing the truth of the statements, which includes their refinement and finding possible connections with similar conditions in the research of other authors, etc.;
7. possible comparison of the results with other known studies on the subject (if possible and available). Finding advantages and disadvantages;
8. creation of appropriate examples illustrating the theory, etc.

There is no doubt in my mind that the above obligations of the doctoral student can without reservation be transcribed as fulfilled by the discussed candidate for the acquisition of the doctoral degree. Another task of mine is to assess the relative share of the PhD student's contribution to this (generally) total work and even more so to the underlying publications. I have no document (protocol, agreement, etc.) that establishes, classifies, or allocates the degree of participation of each of the two co-authors in producing their four joint publications that are presented in this defense. Therefore, I consider the PhD student's participation in reaching the overall results presented to be equivalent to his co-author.

My final opinion on this point of the review is that:

Mira Spasova's contributions to peer-reviewed scientific works are substantial and decisive.

10. Abstract

The abstract fully meets the requirements of the Rules for the Development of the Academic Staff of the PU. It states:

- a brief historical review of the development of the theory of fuzzy sets and the resulting integral equations;
- main objectives of the dissertation work;
- main tasks considered in the dissertation;
- a brief description of the structure of the dissertation work;
- consecutively (following the studies in the dissertation) the main definitions and concepts are given;

- consistently (according to the presentation in the dissertation) the main limitations and the resulting main and auxiliary statements obtained by the doctoral student are formulated;
- basic applications of theoretical results based on concrete examples;
- main conclusions and conclusions arising from the dissertation work, i.e. the doctoral student's contributions;
- author's publications on the topic of the dissertation - 4 publications;
- quotes – 1 quote;
- approval of the obtained results, consisting of:
  - scientific reports at conferences - 5 reports;
  - participation in projects – 3 projects;
- declaration of originality
- bibliography.

Once again, I will point out that the conclusions drawn by the doctoral student correctly reflect what was achieved in the dissertation work proposed for review. In the abstract and in the dissertation, these conclusions are formulated in the section entitled "Conclusion".

The material (in the abstract) is presented in such a way that the reader can gain a complete and adequate idea of the results in the dissertation.

11. Critical remarks and recommendations

I have no significant critical notes and comments that could change my positive opinion of Mira Spasova's dissertation work. Indeed:

- The topic of the dissertation is modern and, so to speak, a "hit" in scientific research on mathematical analysis and integro-differential equations, not only in our country, but also worldwide;
- The specific tasks realizing the goals are chosen correctly and solved sequentially in a natural order and successfully;
- We can 'summarize' the evidence as complete and presented in sufficient academic detail;
- Some concrete implementations of statements are cleverly framed as stand-alone statements in the form of concrete applications;
- Further understanding of the terms and statements presented can be achieved by carefully following the example implementations provided.

As a general recommendation for the design of the dissertation, I will point out that the doctoral student did not clarify to what extent the applied methods and, even more so, the restrictions on the researched objects, where certain qualities of the relevant solutions are fulfilled, were borrowed from other researchers. For example, do the conditions in Theorems 2.1.1 and 2.1.2 occur in analogous studies? This note does not apply only to the two statements cited.

I would allow myself the following recommendation, which reflects my subjective opinion. This means that the recommendation may not be substantial. To do this, let's look at the opening task (A) recorded above in the review. We will note that:

- The derivative of the unknown function is of order k – an arbitrary natural number;
- The integrals involved in writing the equation have both variable and constant boundaries (of the Volterra and Fredholm type);
- The unknown function participates in the integrals non-linearly;
- Nuclei are of the most general kind;
- Finally, let's not forget that some of the constants and functions are fuzzy.

We continue:

- There is hardly any natural or artificial phenomenon that is modeled with such a complex equation. The study of objects "remote" from the needs of people is superfluous. We can represent the working, valuable ideas on relatively simpler equations. Let the user use our ideas, not the resulting formulas - as a reference;
- With so much "inflated complexity" it is possible and permissible to miss the important, conceptual, essence of the working algorithm. As the saying goes: "You can't see the forest from the many trees";
- Some time ago, a teacher of mine (I will spare his name) advised me like this: "Don't examine and prove "everything" on a given topic - leave a little for others as well. Otherwise, how will they quote you."

I think my recommendation is clear, so I won't state it harshly.

The investigations that have been carried out in Section 2.1.4 are concerned only with the uniqueness of the solution, so it is natural that the term "existence" should be dropped from the title of this section.

I noticed ambiguities and technical errors in the text of the dissertation (which are tolerably numerous). I gave them to the author with the sole purpose: If he decides to convert the dissertation into a monographic work, these remarks will be useful. In addition, I consider that the remarks made have no place in the text of the review, since they will in no way change the dissertation - it is a "stone thrown". In addition, it is possible to misrepresent the quality of peer-reviewed research (in other words, to downplay the results), which is unfair.

12. Personal impressions

I have known Mira Spasova for several years. Then we were connected by the common work on the accreditation of doctoral programs in mathematics in several higher schools. After a preliminary meeting and getting to know her, on my recommendation she was selected by the Standing Committee on Natural Sciences, Mathematics and Informatics at the National Agency for Assessment and Accreditation as an expert from the quota of doctoral students in Bulgaria. She did an excellent job with the tasks assigned to her.

My impressions, which I gained during acquaintance with her scientific work, are grounds for asserting the following:

- The doctoral student has appropriate specialized scientific training on the topic of the dissertation work. I will repeat that the topic requires deep specific knowledge in several mathematical disciplines (real mathematical analysis, functional analysis, interval analysis, fuzzy mathematics, integral equations, etc.);

- Mira is a talented and hardworking young mathematician. This combination of qualities, in my opinion, will lead to an upgrade of her achievements, which is a prerequisite for scientific career growth;
- She is diligent and has a "serious interest" in the mathematical objects discussed, as well as in the theories related to them;
- It seems to me that M. Spasova has a need (desire) for work. This can be felt from the way of exposition in the dissertation. This ambition of hers is the "engine" for her growth.

13. Recommendations for future use of the results

It seems to me that I am hardly the most suitable specialist to draw the perspectives and give the guidelines for the development of research in the above-discussed modern and, I would add, quite difficult to achieve results scientific direction. However, I will allow myself to make the following suggestions for possible further research on the subject:

First: One should (if possible and also reasonable) the basic propositions that hold for "traditional" differential, integral, integro-differential and other similar types of equations be "reformulated" and proved for their "fuzzy analogs ". (For brevity, we will refer to the listed classes of equations hereafter as just "equations"). Moreover, we will use the term "traditional" (and also "classical") for equations in the formulation of which the parameters are numerical constants and functions. These reformulated fuzzy analog problems (figuratively speaking) represent a "sea" of statements and unsolved problems. The diversity is due to the following factors:

- the variety of types of classical equations that is generalized;
- the corresponding types of fuzzy equations that represent the generalizations;
- the contents of the tasks that are formulated and solved;
- the metrics that are used in fuzzy analogs, etc.;

Few of these analog problems have been solved. In addition, there is currently no clear classification of the following problems:

- which of the classic problems are directly transferred (with slight additions that have an editorial nature) to the corresponding fuzzy equations;
- which of the tasks require the imposition of additional serious and specific restrictions;
- which known problems cannot be solved with this new class of equations, why, etc.

Second: Specific phenomena that apply only to fuzzy equations should be posited and investigated. This is actually the "important and profitable" research. It is precisely in this type of research that the deep analyzes of this interesting class of equations should be carried out. As an example, I would point to the task:

Problem 1*:* *To find conditions that guarantee that the "thickness" (the distance between the upper and lower functions of the level of the sought fuzzy function, which is a solution of a fuzzy equation of a given type) grows monotonically or strictly monotonically* *with the unbounded increase of time or else (which is more important) decreases or (which is most interesting) this thickness tends to 0.*

The choice of distance (metric) in the above task is also a discussion task. The formulated task "is not sucked from the fingers". It seems to me that it lies at the heart of successful robustness studies of fuzzy equation solutions. Indeed, when studying some types of decision persistence, it is usually expected that they (the decisions) tend to 0 with an unbounded increase in the independent variable (usually this variable is taken to be time). If the thickness of the fuzzy function grows with time this "expectation" will not happen.

Third: The fundamental and qualitative theory of fuzzy equations with impulse effects should be established. Subsequently, of course, approximate methods for finding the solutions of such classes of equations. As an illustrative example, we will consider a fuzzy impulse Volterra model at fixed instants of impulse action. The fuzzy model (without impulses) was considered by the PhD student (see page 35 of the thesis, equation (1.5.2)). The impulse model (classical or fuzzy) has the form:

$$\begin{aligned} &\frac{dP}{dt} = aP - bP^2 - cP\int_0^t P(\tau)d\tau, \quad t \neq t_i; \\ &\Delta P(t_i) = P(t_i + 0) - P(t_i) = I_i,\ i = 1,2,\ldots; \\ &P(0) = P_0, \end{aligned} \tag{P}$$

where:

- $P = P(t)$ is the amount of the population at time t≥0;
- a is the birth rate;
- b is the crowding factor;
- c is the toxicity coefficient;
- P_0 is the initial amount of the population;
- $t_1, t_2, \ldots,\ \ 0 < t_1 < t_2 < \cdots$, are fixed moments of impulse impacts,
- $I_1, I_2, \ldots$ are the magnitudes of the impulse effects.

It is clear that if $I_i > 0$, then at time t_i we have biomass addition to the population and if $I_i < 0$, then biomass is taken away at that time. In the general case, it is impossible to obtain the exact values of the parameters $a, b, c, P_0, I_1, I_2, \ldots$ of the model, but it is possible to determine their permissible limits of variation. Then it is natural to assume that the specified parameters are fuzzy numbers, i.e. belong to E^1, their r levels being respectively:

$$a(r) = [a_1, a_2];\ b(r) = [b_1, b];\ c(r) = [c_1, c_2];\ P_0(r) = [P_{01}, P_{02}];\ I_i(r) = [I_{i1}, I_{i2}], i = 1,2, \ldots$$

Therefore, the model (P) described above can generally be considered as a fuzzy impulse integro-differential equation with fixed moments of impulse action. I will mention that even more general is the model when the impulse moments are variable. For such and similar equations, the fundamental theory is in the initial stage of construction, and their qualitative theory has not been conceived. The corresponding "sea" of problems for the described equations (in which there is a place for all Chinese) first of all includes the questions of existence, uniqueness, form of their solutions and other analogous questions. Obtaining sufficient conditions establishing traditional properties of the solutions, such as: continuous dependence, monotonicity, boundedness, robustness, asymptotic robustness, robustness to persistent disturbances, etc. are the next group of tasks to be solved. Here I will take the liberty of formulating the following specific task:

Problem 2: We consider (P) when $c = 0$. In this case, the model obtained from (P) is called the Verhulst logistic model. To find conditions for periodicity of the solution.

It seems to me that it will turn out that for periodic solutions to exist the impulse effects $I_1, I_2, \ldots$ must be real constants (not fuzzy numbers).

Fourth: Here I am tempted to point out (in my opinion) the most attractive group of tasks. To this group I include the tasks of finding conditions for the existence of specific qualities of solutions resulting from impulse effects. As examples I will indicate: continuous dependence and stability of solutions of fuzzy impulse equations from:

- the impulse moments;
- the magnitudes of impulse effects;
- the impulse hypersurfaces (for some classes of impulse fuzzy equations with variable momentum moments), etc.

In order not to be "vulgar" I will formulate one such specific task (in a narrative style). For this purpose I will again use the model (P). Consider the perturbed (P) fuzzy impulse initialization problem:

$$\begin{aligned} &\frac{dQ}{dt} = aQ - bQ^2 - cQ\int_0^t Q(\tau)d\tau, \quad t \neq \theta_i; \\ &\Delta Q(t_i) = Q(\theta_i + 0) - Q(\theta_i) = I_i; \\ &Q(0) = P_0, \end{aligned} \tag{Q}$$

where the angular momentums are $\theta_1, \theta_2, \ldots,\ 0 < \theta_1 < \theta_2 < \cdots$. The differences between tasks (P) and (Q) are only the impulse moments, i.e. we can assume that in the basic problem (P) the momenta are perturbed, resulting in problem (Q). We will say that the solution of the fundamental problem (P) depends continuously on the angular momentum if:

$$\begin{aligned} &(\forall \varepsilon = const > 0)(\forall T = const > 0)(\exists \delta = \delta(\varepsilon, T) > 0): \\ &(\forall \theta_1, \theta_2, \ldots,\ 0 < \theta_1 < \theta_2 < \cdots;\ |\theta_1 - t_1| < \delta,\ |\theta_2 - t_2| < \delta, \ldots) \\ &\Rightarrow \rho\big(\{(t, P(t));\ 0 \le t \le T\},\ \{(t, Q(t));\ 0 \le t \le T\}\big) < \varepsilon. \end{aligned} \tag{R}$$

In the definition expression (R) the symbols are used: using $G_P = \{(t, P(t));\ 0 \le t \le T\}$ and $G_Q = \{(t, Q(t));\ 0 \le t \le T\}$ denote the graphs of the fuzzy functions $P(t)$ and $Q(t)$ in the interval $[0, T]$, respectively; ρ denotes a suitable metric between the graphs G_P and G_Q. For example, the chosen metric may be Hausdorff. In this case, I will note that since the fuzzy functions $P(t)$ and $Q(t)$ are discontinuous at the points of impulse impacts, finding the Hausdorff distance between their graphs is a "thin" task that requires extensive preliminary research. We can now formulate the corresponding problem arising from the definition (R) as follows:

Problem 3: *To find sufficient conditions under which the solution of a fuzzy impulse initialization problem for a given class of equations depends continuously on the impulse moments.*

I will say: what a simple and elegant formulation of Problem 3, and what "trouble" is required to solve it. I will draw attention to one more specific feature of the fuzzy momentum equations. Since the impulse effects essentially represent the addition of fuzzy numbers to the studied solution (fuzzy function) at certain moments, after each impact the "thickness" of the solution increases impulsively (instantly, in leaps). An increase in the thickness of the solution in a global sense (for example at $t\epsilon[0,T]$, where T is a relatively large positive constant) is inadmissible in studies of continuous dependence or stability of solutions (in the second case $T=\infty$). Therefore, it is imperative to propose such conditions, in the presence of which the thickness of the solution monotonically decreases the "continuous part" of the solutions, i.e. between impulse moments. In this way, it is possible to compensate for the "diffusion" of the lower and upper level functions in a global sense.

Fifth: A relatively complete characterization should be made of the working approximate methods for finding the solutions of fuzzy equations, which are essentially analogs of the popular methods designed for the classical equations. The fuzzy equations studied should be significantly simpler and class-sorted than those considered in the dissertation. The characterization should include: which of the classical methods for approximating the solutions remain valid for the solutions of the corresponding fuzzy equations; what additional conditions and why they should be required; does the class of fuzzy equations for which a given approximation method is valid narrow; do additional difficulties arise with fuzzy equations, etc.

Sixth: The introduction and study of specific models from various sciences (not only natural and engineering sciences, but also economic, social, etc.), and also the modeling of objects from practice through fuzzy equations will undoubtedly raise modeling to a higher level . This is, on the one hand, and on the other hand, it will help the emergence of new theoretical tasks, which will lead to the introduction and study of new classes of model fuzzy equations.

Conclusion

The notes and analyzes made above in the review give me reason to draw the following general conclusions:

1. Scientific works (dissertation and relevant scientific publications) contain new theoretical studies. The obtained results develop and enrich mathematical knowledge. They are an original contribution of the candidate for obtaining the educational and scientific degree "doctor";
2. The obtained results are closely related to the mathematical analysis, i.e. correspond to the doctoral program on which the dissertation was presented;
3. The doctoral student's research has been published in reputable publications (AIP Conference Proceedings), which are reflected in the Web of Science and Scopus databases;

4. The candidate's achievements fulfill, and in some indicators significantly exceed the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB) and the Regulations for the Implementation of ZRASRB regarding the acquisition of the educational and scientific degree "doctor";
5. I did not detect plagiarism.

On the basis of the above, I give my positive assessment of the conducted research, presented in the above-reviewed dissertation work, abstract and scientific publications. I propose to the Scientific Jury to award the educational and scientific degree "doctor" to Mira Lachezarova Spasova on:

- Field of higher education: 4. Natural sciences, mathematics and informatics;
- Professional direction: 4.5. Mathematics;
- Doctoral program: Mathematical analysis.

05.04. 2024

Reviewer: ....

(Prof. DSc Angel Dishliev)