

OPINION

by **Prof. Dr. Hristo Stefanov Kiskinov, PhD,**
Professor at Plovdiv University "Paisii Hilendarski" (PU),
Faculty of Mathematics and Informatics

of a dissertation for the award of the educational and scientific degree "**Doctor**" (**PhD**)
by: Area of Higher Education 4. *Natural Sciences, Mathematics and Informatics*;
Professional Field 4.5. *Mathematics*;
Doctoral Program *Mathematical Analysis*.

Author: Mira Lachezarova Spasova.

Title: "Analytical Methods for Solving Some Classes of Fuzzy Integro-Differential Equations".

Scientific Supervisors: Prof. Dr. Atanaska Tencheva Georgieva.

1. General Presentation of the Procedure and the PhD Student

By order No. PD-21-454 from 23.02.2024 of the Rector of the Plovdiv University "Paisiy Hilendarski" (PU), I have been appointed as a member of the scientific jury to ensure a procedure for the defense of a dissertation (PhD-thesis) titled "Analytical Methods for Solving Some Classes of Fuzzy Integro-Differential Equations" for awarding the educational and scientific degree "doctor" in the area of higher education 4. *Natural sciences, mathematics and informatics*, professional field 4.5 *Mathematics*, doctoral program *Mathematical Analysis*. The author of the dissertation is Mira Lachezarova Spasova - full-time PhD student at the Mathematical Analysis Department of the Faculty of Mathematics and Informatics (FMI) at Plovdiv University Paisiy Hilendarski with scientific supervisors Prof. Dr. Atanaska Tencheva Georgieva.

The set of paper and electronic materials presented by the PhD student Mira Lachezarova Spasova is in full compliance with Article 36 (4) of the Regulations for the Development of the Academic Staff of the PU and includes all the necessary documents.

Mira Lachezarova Spasova was born on 19.02.1995. In 2018, she obtained the bachelor's degree at the FMI of PU "Paisiy Hilendarski" with the professional qualification "Teacher of Mathematics, Informatics and Information Technologies", and in 2019 the educational degree "Master's" again at PU "Paisiy Hilendarski" with professional qualification "Mathematics". Since 2020, she has been a full-time doctoral student at the Department of Mathematical Analysis at the FMI at the PU. Since 2018, she has been working as an informatics and information technology teacher at the Sports School in Pazardzhik.

2. Actuality of the Research Topic

Fuzzy differential and integral equations are a powerful tool for modeling dynamic systems describing processes and phenomena from mathematical physics, financial and economic systems, and financial mathematics. Characteristic of these models is the recognition that the data may not be precisely determined, that there may be measurement errors, or that the data may be insufficient. The topic is particularly relevant nowadays, which is easily established by following the publication activity of those working on this topic in specialized journals.

3. Knowing the Problem

The carried out review of the scientific research on the problem, the selected literature and above all the obtained results speak unequivocally of deep knowledge in the researched field.

4. Research Methodology

The methodology of the research is standard for most mathematical studies – proving statements, constructing methods based on the proven statements, their numerical implementation and presentation of illustrative examples.

5. Characterization and Evaluation of the Dissertation Work and Contributions - Presence/Absence of Plagiarism

The dissertation (PhD-thesis) "Analytical Methods for Solving Some Classes of Fuzzy Integro-Differential Equations" is dedicated to finding exact and approximate solutions of some classes of fuzzy integro-differential equations by using analytical methods. It is written in 107 pages and consists of an introduction, four chapters, a conclusion and a bibliography with 103 titles. The exposition also includes a useful list of used symbols and abbreviations, 9 graphs and 1 table. In the introduction, in addition to formulating the goals and tasks, some more significant results related to the fuzzy theory of integral and integro-differential equations are marked, which are directly related to the topic of the present dissertation work. The first chapter has an overview character and it presents definitions and properties of the main concepts that are used later in the dissertation work. Chapters 2, 3 and 4 describe the conducted research and analysis, and the main obtained scientific and scientific-applied results. In the conclusion, the doctoral student made a self-assessment of the contributions contained in the dissertation and described the approbation of the obtained results.

I support the main contributions described by the PhD student in the current dissertation. Namely:

- Sufficient conditions for the existence and uniqueness of the solution of a nonlinear fuzzy Volterra-Fredholm integro-differential equation are found.

- A fuzzy analytical method using the Adomian decomposition method is constructed to find the approximate solution of a nonlinear fuzzy Volterra-Fredholm integro-differential equation. Sufficient conditions for the convergence of the method are found and an estimate of the error is obtained.

- A fuzzy transform of Sumudu is constructed. Sufficient conditions for the existence of the transformation and its application to ordinary and partial fuzzy derivatives are found.

- A fuzzy analytical method is constructed which is a combination of Sumudu fuzzy transform and Adomian decomposition method to find the approximate solution for the nonlinear Volterra-Fredholm fuzzy integrodifferential equation.

- A fuzzy Natural transform is constructed to find the exact solution of a linear fuzzy Volterra integro-differential equation with a convolutional kernel. Sufficient conditions for the existence of the transformation and its connection with the Laplace and Sumudu transformations are found.

- A fuzzy analytical method is constructed that uses the fuzzy variant of the Sumudu transform to find the exact solution of a linear partial fuzzy Volterra integro-differential equation.

- A fuzzy two-dimensional Natural transform is constructed to find the exact solution of a linear partial fuzzy Volterra integro-differential equation. Sufficient conditions for the existence of the transformation and its application for fuzzy partial derivatives are found.

I do not detect "plagiarism" in the works of the author and the presented thesis in the sense of the Law on the Development of the Academic Staff in the Republic of Bulgaria.

6. Assessment of the Dissertation's Publications and Personal Contributions of the Author

The presented dissertation is based on 4 publications in English, all in the journal AIP Conference Proceedings, indexed in Web of Science, SCOPUS and with SJR=0.189 (2021) and SJR=0.164 (2022). The articles are based on corresponding reports in traditional international scientific conferences held in Sozopol (AMEE'20, AMEE'21) and in St. Constantine and Elena (NTADES'20, NTADES'21). They form a total of 120 points, which exceeds exactly four times the minimum national criteria for this indicator, requiring 30 points. All 4 presented articles are co-

authored with the scientific supervisor. For me, the candidate's personal contribution to them is undoubted.

I have no major criticisms. But I would still recommend a greater variety of places to publish in the future.

7. Summary

The Summaries, written in Bulgarian and in English, have 32 pages, correspond to the requirements of RDASPU and contain the main results obtained in the dissertation work.

8. Recommendations for Future Use of Dissertation Contributions and Results

I wish the PhD student to continue working in this interesting field and with the same enthusiasm.

CONCLUSION

The dissertation *contains scientific, scientific-applied and applied results, which are an original contribution to the science* and **meet all the requirements** of the Law on the Development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for the Implementation of the LDASRB and the relevant Regulations of Plovdiv University "Paisii Hilendarski". I detect no plagiarism. The presented materials and dissertation results far exceed the minimum national requirements introduced by the Regulations for the Implementation of the LDASRB.

The dissertation work shows that the PhD student Mira Lachezarova Spasova possesses in-depth theoretical knowledge and professional skills in the scientific specialty Mathematical Analysis, demonstrating qualities and skills for conducting research with obtaining original and significant scientific contributions.

Due to the above, I confidently give my **positive assessment** of the conducted research, presented by the above-reviewed PhD thesis, summary, achieved results and contributions, and ***I propose to the honorable scientific jury to award the educational and scientific degree "Doctor" (PhD)*** to Mira Lachezarova Spasova in the Area of Higher Education 4. *Natural sciences, Mathematics and Informatics*; Professional Field 4.5. *Mathematics*; Doctoral Program *Mathematical Analysis*.

08.04.2024

Plovdiv

Scientific jury member:

Prof. Dr. Hristo Stefanov Kiskinov