

OPINION

by Prof. Dr. Mihail Mihaylov Konstantinov

Department of Mathematics
University of Architecture, Civil Engineering and Geodesy - Sofia

under a procedure for awarding the educational and scientific degree
PhD

PhD student: Mira Lachezarova Spasova

Thesis topic: Analytical methods for solving some classes of fuzzy integro-differential equations

Field of higher education: 4. Natural Sciences, Mathematics and Informatics

Professional field: 4.5 Mathematics

Doctoral program: Mathematical Analysis

Scientific supervisor: Prof. Dr. Atanaska Tencheva Georgieva

1. Information on the procedure

By Order RD-21-454 of 23.02.2024 of the Rector of the Plovdiv University "Paisii Hilendarski" I was appointed as an external member of the Scientific Jury under the procedure for acquiring the educational and scientific degree "Doctor" by Mira Lachezarova Spasova. At the first meeting of the Jury, I was appointed to prepare an opinion on the procedure. At this meeting it was established that the candidate has 188 points according to the criteria of NACID with 80 points required.

The materials of the procedure were provided to me in electronic form. I confirm that these materials meet the requirements of the Law on the Development of Academic Staff in the Republic of Bulgaria and the normative

documents of Plovdiv University “Paisii Hilendarski” on the awarding of scientific degrees.

2. Description of the dissertation

The dissertation is written on 108 pages and is divided into an overview and three chapters on the merits. Publications and citations on the subject of the work are described. A declaration of originality is presented according to the requirements of the Law. The literature includes 103 titles, of which 101 in English and two in Russian.

3. Relevance of the dissertation

Fuzzy differential and integro-differential equations are used to model a number of processes in engineering, computer engineering and communications, artificial intelligence, etc. In particular, these equations are widely applied in modern control theory. Therefore, I believe that the topic of the dissertation is relevant and promising.

4. Brief description of the dissertation

The review presents brief information on fuzzy sets and fuzzy functions of one and two variables, including fuzzy derivatives and fuzzy integrals. A fuzzy population model of Volterra is also considered. In Ch. 2 decomposition methods for the solution of nonlinear fuzzy integro-differential Fredholm-Volterra equations are discussed. In Ch. 3 and 4 fuzzy transformations have been considered for solving linear fuzzy integro-differential equations of Fredholm-Volterra type. The contributions in the dissertation are correctly formed, and the Autoreferate corresponds to the content of the dissertation.

5. Description of contributions

The main contributions of the dissertation are as follows.

- Sufficient conditions for existence and uniqueness of the solutions of the considered classes of fuzzy integro-differential equations were found.
- Efficient fuzzy methods and schemes for solving these equations have been constructed, and both approximate and exact schemes have been considered.

6. Publications on the dissertation

The work is accompanied by a list of 4 publications (co-authored with the supervisor) in the prestigious edition “Proceedings of the American Institute of Physics”, which has a high SJR. There is evidence of a citation of one of the candidate's works.

7. Technical remarks on the dissertation

- In the definition of \mathbf{R}_+ on p. 4 the word ”positive” should be replaced by “non-negative”.
- On p. 25 there is an error from the automatic citation in TeX.
- In the literature it is good to add the numbers ISBN and doi where possible.
- In the literature, the titles of the articles are given in *Italic* and the names of the sources in Roman. It's usually the other way around. Also, the names of the authors are given in **Bold**, while according to the Bulgarian standard it is accepted that they are in Roman.
- I found a systematic error in the spelling of the word Engineering.

8. Recommendations on the dissertation

I recommend to consider developing efficient numerical methods and algorithms for solving fuzzy equations of the classes considered, including with consideration of the effects of finite machine arithmetic.

Fruitful may be the idea of modelling machine arithmetic with floating point by fuzzy numbers and fuzzy functions. I recommend trying to develop this idea (in parallel with the ideas and methods of interval arithmetic), which can lead to significant application effects.

I recommend that the dissertation work be expanded (including with more examples) and be formed as a monograph in Bulgarian or English. This will be beneficial for the candidate's further professional development.

9. Conclusion

The dissertation work is well written and contains enough scientific and applied scientific contributions to the theory of fuzzy differential and integro-differential equations. It meets the requirements of the Law on the Development of Academic Staff in the Republic of Bulgaria and the Regulations for its application, as well as the Rules of Procedure of Plovdiv University “Paisii Hilendarski” on the awarding of academic degrees and the granting of academic positions.

Therefore, my opinion on the dissertation is POSITIVE and I strongly recommend to the Venerable Scientific Jury to award to the candidate Mira Lachezarova Spasova the educational and scientific degree “Doctor” in the specified field of higher education and professional field.

Sofia, 26.03.2024

Signed by:

(Prof. M. Konstantinov)