REVIEW

by Professor DSci. Veseilin Totev Videv

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Member of the scientific jury, regarding the competition for the academic position of “associate professor”, field of higher education 4. “Natural sciences, mathematics and informatics”, professional field 4.5 Mathematics (Geometry and topology)

1. Presented materials. In the competition for "Docent", announced in the State Gazette, issue 96 of 17.11. 2023 and on the website of Plovdiv University "Paisiy Hilendarski", for the needs of the "Algebra and Geometry" department at the "Mathematics and Informatics" Faculty, as the only candidate participated Iva Rumenova Dokuzova, Ph.D. from FMI, PU . By order No. RD-21-387 of 02/16/2024, of the Rector of PU, I was appointed as a member of the scientific jury and after the first meeting of the jury on 02/22/2024, I was voted a reviewer. The one presented by Ch. Assistant Professor Iva Rumenova Dokuzova, Ph.D. a set of materials on paper is in accordance with the Rules for the Development of the Academic Staff of the PU, and includes the following documents:

1. Application form to the Rector for admission to participate in the competition;
2. Resume in European format;
3. Diploma of higher education with acquired educational and qualification degree "master" - original with attachment or notarized copy;
4. Diploma for educational and scientific degree "doctor"
5. List of scientific and educational works;
6. Scientific and educational works;

6.1. List of citations;

7. a) Certificate of compliance with the minimum national requirements;

   b) Certificate of compliance with additional faculty requirements;

8. Annotations of the materials under Art. 65. from PRASPU;
9. Self-assessment of contributions;
10. Declaration of originality and authenticity of the attached documents;
11. Certificate of work experience;
12. Documents for academic work;
13. Documents for scientific research work;
14. Documents for administrative work.

2. Brief biographical data. Iva Rumenova Dokuzova, PhD, was born on January 12, 1971 in the city of Sofia. He received his secondary education at "Georgi Dimitrov" ESPU, "Konstantin Velichkov" ESPU, TOH "Asen Zlatarov", Plovdiv in the period 1985-1989. In the period 1989-1994 he graduated from the Faculty of Mathematics and Informatics of Plovdiv University "Paisiy" Hilendarski", specialty "Mathematician and teacher of mathematics". In the period 2003-2006, he was a doctoral student in the Department of Algebra and Geometry of the FMI, PU, defending his doctoral thesis on the topic "*On the geometry of almost complex manifolds with Norden metric and Riemannian manifolds with almost product structure*" and obtained the degree "Doctor of Geometry and Topology". In the period 1995-1996, he was a Mathematics teacher at Nikola Vaptsarov Secondary School, Samokov. In the period 1996-1998 he was an assistant, in the period 1998-2001 he was a senior assistant, and in 2001-2009 he was the chief assistant in the "Lyuben Karavelov" branch of the PU "Paisiy Hilendarski", the city of Kardzhali. From 2009 until now, he has been the chief assistant in the "Algebra and Geometry" department of FMI of PU "Paisiy Hillen-darski" and secretary of the department since 12.01.2024.

3. General characteristics of the applicant's activity.

3.1. Assessment of educational and pedagogical activity. The candidate leads lectures and seminars in the following disciplines: Synthetic Geometry, Descriptive and Differential Geometry, Linear and Analytical Geometry. She has developed 6 new lecture courses in the last five years: "Mathematical foundations of computer graphics" - an optional subject for all undergraduate majors in the Faculty of Mathematics and Informatics (FMI); "Geometry" for the specialty "Mathematics, Informatics and Information Technologies" and the specialty "Information Technologies, Mathematics and Educational Management" at the Faculty of Mathematics and Informatics; "Discrete Geometric Structures" for the "Mathematics" major and the "Applied Mathematics" major at the Faculty of Mathematics and Informatics; "Latech Publishing System" for the "Mathematics" major and the "Business Mathematics" major at the Faculty of Mathematics and Informatics; "Models of random processes" for the specialty "Biostatistics", joint master's program of PU "Paisiy Hilendarski" and MU-Plovdiv; "Data cluster analysis" for the specialty "Biostatistics", joint master's program of PU "Paisiy Hilendarski" and MU - Plovdiv; "Self-learning systems" for the specialty "Biostatistics", joint master's program of PU "Paisiy Hilendarski" and MU - Plovdiv. She prepared 4 electronic resources for studying students and two textbooks, she is the scientific supervisor of two graduates.

3.2. Evaluation of the candidate's scientific and scientific-applied activity.

3.2.1. Compliance with minimum national requirements. According to the report on compliance with the minimum national requirements of the RASRB in District 4. Natural sciences, math-

ematics and informatics, professional direction 4.5 Mathematics, head of the program Dr. Iva Dokuzova has 606 points on the relevant indicators, with a required number of 400 minimum required points for the academic position “associate professor”.

3.2.2. Compliance with the additional requirements of FMI (According to Art. 66 (2) item 7 of the PRASPU).

3.2.2.1. At least 8 publications must be submitted, which should not be submitted for the ONS "Doctor". One monograph and 15 publications were submitted for the competition, none of which were used in the dissertation work for the ONS "Doctor", defended on 10.03.2006.

3.2.2.2. Of the submitted publications, at least 5 must be in journals. Of the presented journal publications, the following 11 works are: [1], [2], [6-10] and [12-15].

3.2.2.3. Of the submitted publications, at least 3 must be in journals with an impact factor. Among the presented publications with an impact factor are works [10], [13] and [15], and the monograph is equated to 2 publications with an impact factor.

3.2.2.4. At least 1 textbook or 1 study aid must be submitted. 2 textbooks are presented.

3.2.2.5. Requirement. Evidence of at least 5 citations. 41 citations are noted (without self-citations).

3.2.3. Characteristics of the candidate's scientific works. The candidate, Assistant Professor Iva Rumenova Dokuzova, PhD, submitted a total of 22 scientific works, 1 monograph, 2 textbooks. 15 non-dissertation scientific papers, 2 study aids and 3 research projects are accepted for review and are counted in the final assessment. 7 scientific papers, which are related to the dissertation and outside the scope of the competition, are not peer-reviewed. Of the scientific publications, 9 are independent, 2 are with one co-author and 4 are with two co-authors, 11 are in journals (8 abroad) and 4 are in works at scientific conferences. Three of the publications have an impact factor, 6 publications are with SJR, 7 publications are indexed in Web of Science and Scopus, 10 publications are indexed in Zentralblatt and MathSciNet. We will separately consider the scientific, scientific-applied and applied contributions of the scientific articles presented by the candidate. Many geometric studies are devoted to differentiable manifolds equipped with a pseudo-Riemannian (Riemannian) metric and an additional tensor structure. In modern differential geometry, fundamental tensor structures of type (1,1) are the almost product structure. The subject of study in the articles [1], [2] and [3] are differentiable manifolds equipped with such structures. In [1], a symmetric affine connection $\bar{\nabla}$ was defined in a manifold from the class of non-integrable Riemannian manifolds with an almost product structure J. Conditions under which $\bar{\nabla}$ is an equi-affine connection were found. In [3], a Riemannian manifold of local decomposition was characterized, i.e. the structure J satisfies $\nabla J=0$. In publication [2], the class of quasi-Keller manifolds with an almost complex struc-

ture and Norden metric is characterized, a symmetric affine connectivity in a manifold of this class is defined by the Riemannian connectivity ∇ and two smooth vector fields. Conditions under which $\bar{\nabla}$ is equiaffine connectivity are found. Special cases are considered where, if $\nabla J=0$ and $\bar{\nabla}$ is a locally Euclidean connectivity, then ∇ is also locally Euclidean. In works [4], [5], [6], [10], [12], [13] and [15] three-dimensional Riemannian manifolds with a circulant metric and with an additional circulant structure q as the third power of the structure were characterized is equal to the identity. We will note that K.Yano considers manifolds in which the additional structure satisfies an equation of the third degree. In publication [6], a similar problem was solved for three-dimensional manifolds. A necessary and sufficient condition is found for the structure to be parallel with respect to Riemannian connectivity. In work [4] an almost conformal image of the metric g for a manifold $(M\ g,q)$ is defined. In [5], a particular case of an almost conformal image of the metric g for a manifold $(M\ g,q)$ which is of parallel structure q with respect to the Riemannian connectedness of g is given. In work [10], three classes of manifolds $(M\ g,q)$ are considered, for which the curvature tensor R is invariant with respect to the additional structure q. In [12] the components of the curvature tensor for the Riemannian connectivity of a manifold $(M\ g,q)$ were calculated. In [13], the adjoint metric $\tilde{g}=g(x,qy)+g(qx,y)$ was considered and a fundamental tensor F on the manifold $(M\ g,q)$ was defined using the covariant derivative of $\tilde{g}$. In [15], the manifold $(M\ g,q)$ and the adjoint manifold $(M,\tilde{g},q)$ were studied. In work [8], a three-dimensional Riemannian manifold M with an additional tensor structure q, whose third power is minus the identity, was introduced. The matrices of the components of q and of the metric g are circulant. A necessary and sufficient condition is obtained for the structure q to be parallel with respect to the Riemannian connectivity generated by g. An example of the considered variety is given. In works [7], [9], [11] and [14], four-dimensional Riemannian manifolds with a circulant metric and an additional circulant tensor structure whose fourth degree is the identity are considered. The geometric properties of the manifolds and how the circularity of the components of the structures affects these properties are investigated. These four works are part of the presented monograph. In the first part of the monograph, a four-dimensional Riemannian manifold (M,g,Q) is presented, equipped with an additional tensor structure Q, whose component matrix is circulant and the fourth degree of Q is the identity. The manifold (M,g,Q) was introduced for the first time by D.Razpopov. In Section 4, an adjoint metric $\tilde{g}$ is introduced on (M,g,Q) and an almost conformal image $\alpha g+\beta\tilde{g}$ is defined. Conditions for the parallelism of the additional structure Q with respect to the connectivity generated by the nearly conformal image are found. Section 5 considers manifolds (M,g,Q) whose curvature tensors are invariant with respect to Q. Section 6 continues the study of (M,g,Q) and its associated manifold (M,g,P) , according to the classifications of A. Naveira, G. Ganchev, M. Staykova and K. Gribachev. Some geometric charac-

teristics of the built examples were obtained. In the second part of the monograph, a four-dimensional Riemannian manifold M, equipped with an additional tensor structure S, whose fourth power is minus the identity, is studied. In paragraph 12, manifolds (M,g,S) whose curvature tensors are invariant with respect to S are considered. In paragraph 13, a necessary and sufficient condition for the structure S to be parallel with respect to Riemannian connectivity of the metric g is obtained. Examples of the considered manifolds on Lie groups have been constructed and some of their geometric characteristics have been obtained. In § 14 an equation of a central hypersphere in TpM is found in terms of the adjoint metric $\tilde{g}$. Spheres with respect to $\tilde{g}$ in special three-dimensional subspaces of TpM are considered and their equations are derived. Some special sites in TpM are also considered and the equations of circles with respect to $\tilde{g}$ in these sites are obtained. All equations of the studied curves and surfaces are interpreted in terms of g. In the works of the candidate, deep and meaningful results of differential geometry were obtained on a topic started by the great Russian scientist A.Norden, continued by K.Gribachev, D.Mekerov, M.Manev, G.Ganchev, G.Dzhelebov and others of ours scientists.

3.2.4. Evaluation of the candidate's scientific activity. General Assistant Dr. Iva Dokuzova has participated in 3 scientific projects of international and national nature. He is a member of the Union of Mathematicians in Bulgaria and the Geometric Society "Boyan Petkanchin. She has presented 17 scientific reports at university, faculty, regional, national and international conferences and congresses (10 of the listed conferences are international). She has a diploma of honor for the support provided to the activities of FMI on the occasion of the anniversary "40 years of FMI at PU", Plovdiv, 2010. She was awarded an honorary plaque for the support provided to the activities of FMI on the occasion of the anniversary "40 years of FMI at PU", Plovdiv, 2010. Plovdiv, 2010.

3.2.5. Quoted. There are 41 citations of the candidate's scientific works in recognized scientific journals referenced and indexed in Scopus, Web of Science and Zentralblatt Math, by well-known domestic and foreign authors, which confirms the importance of scientific and production.

4. Evaluation of the candidate's personal contribution. The personal contribution of the candidate in the respective scientific publications is indisputable as 9 of the scientific articles are independent, in 3 scientific articles she is the first author, in 2 scientific articles she is the second author and in 1 scientific article she is the third author.

5. Evaluation of the candidate's administrative activity. The administrative activity of the candidate is reduced to participation in the organization of the educational activity of the joint master's program "Biostatistics" at the Polytechnic "Paisiy Hilendarski" and the Medical University-Plovdiv. Member of departmental committees for adapting the study programs in "Geometry" (2 study programs 2021/22 academic year), Linear Algebra and Analytical Geometry. (2 study pro-

grams 2021/22 academic year), Synthetic Geometry (1 study program 2021/22 school year), "Elements of combinatorics, probability theory and mathematical statistics at school" OMIIT 1 year and 2 year (2 study programs 2021/ 2022), Latech Publishing System (1 curriculum 2021/2022). Participation in the committee for checking science reports and accreditation documents in the department from 2013 to the present. Member of the committee for the State Examination in Mathematics from the 2012/2013 school year until now. Member of the accreditation committee at FMI from the 2020/2021 academic year. Secretary of the "Algebra and Geometry" department from 17.11.2015 until now. Participation in KSK and organization of celebrations and promotions. Participation in the committee for checking the mathematics exam in KSK'2019, KSK'2020, KSK'2021, KSK'2022, KSK'2023. Participation in the commission for the organization and holding of the solemn promotions for the awarding of diplomas to the graduating bachelors and masters at FMI (Graduation'2016, Graduation'2017, Graduation'2018, Graduation'2019, Graduation'2022).

6. Critical remarks and recommendations. I have no critical comments on the materials presented by Dr. Iva Dokuzova. I also recommend that he continue his scientific activity in the subject he has been working on so far.

7. Personal impressions. My personal impressions of the candidate are excellent, both as a teacher and as a researcher.

8. Conclusion. The documents and materials presented by Ch.assistant professor Iva Rumenova Dokuzova, PhD, meets all the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), the Regulations for the Implementation of ZRASRB and the relevant Regulations of the PU "Paisiy Hilendarski". The candidate in the competition has submitted a sufficient number of scientific works published after the materials used in the defense of the "Doctor" and before submitting the documents in the current competition. The candidate's works contain original scientific and applied contributions that have received international recognition, a representative part of which has been published in journals and scientific collections issued by international academic publishing houses. The theoretical and developments have practical applicability, and some of them are directly oriented to the academic work. The scientific and teaching qualification of Ch. Assistant Professor Iva Rumenova Dokuzova is unquestionable. The achievements of Ch. Assistant Professor Iva Rumenova Dokuzova, Ph.D. results in educational and scientific research activities fully correspond to the minimum national and additional requirements of the Faculty of "Mathematics and Informatics", adopted in connection with the Regulations of the PU for the application of ZRASRB.

All this gives me reason, convinced, to give my **positive assessment** of the candidacy of assistant professor Dr. Iva Rumenova Dokuzova for taking the academic position “associate professor” in professional direction 4.5-Mathematics (Geometry and topology).

Date: 22.03.2024 Member of the scientific jury:…….

(Prof. DSci. Veselin Videv)