OPINION

By Associate Professor Marta Kostadinova Teofilova, PhD,

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of the materials submitted for participation in the competition
for the academic position of **Associate Professor**
**at the Faculty of Mathematics and Informatics,
University of Plovdiv Paisii Hilendarski**

In the field of higher education 4. *Natural sciences, mathematics and informatics*
professional direction 4.5. *Mathematics*
scientific specialty *Geometry and Topology.*

In the competition for the position of **Associate Professor,** announced in the State Gazette, no. 96 of 17.11.2023 and on the Internet page of the University of Plovdiv Paisii Hilendarski for the needs of the Department Algebra and Geometry at the Faculty of Mathematics and Informatics, as the only candidate participates Chief Assistant Iva Rumenova Dokuzova, PhD from the Department of Algebra and Geometry Department, Faculty of Mathematics and Informatics.

1. General Presentation of the Procedure and the Candidate

By order No. RD-21-387 dated February 16, 2024 of the Rector of University of Plovdiv Paisii Hilendarski (PU), I have been appointed as a member of the scientific jury of the competition for the academic position **Associate Professor,** in the field of higher education 4. *Natural Sciences, Mathematics and Informatics,* professional direction 4.5. *Mathematics* (scientific specialty *Geometry and Topology*), **announced for the needs of the Department of Algebra and Geometry** at the Faculty of Mathematics and Informatics.

Only one candidate has submitted documents for participation in the competition: Chief Assistant Professor Iva Rumenova Dokuzova, PhD from the Department of Algebra and Geometry at the Faculty of Mathematics and Informatics, PU.

The presented by Chief Assistant Iva Dokuzova set of materials on paper and electronic media is in accordance with the Rules for the Development of the Academic Staff of the University of Plovdiv. The candidate has submitted a total of 15 scientific papers, 1 monograph and 2 textbooks and manuals for students. Sixteen scientific works (15 papers and 1 monograph) are accepted for review, which are outside of the doctoral dissertation. In the final evaluation, 2 textbooks for students and participation in 3 scientific research and educational projects (2 university projects and 1 international project) will be counted. Seven scientific works on the PhD thesis are not reviewed (presented in a separate list). The distribution of scientific works by relevant categories, in the country and abroad, is as follows: 7 publications and 1 monograph are in Bulgarian peer-reviewed journals or conference proceedings, and 8 publications are in international peer-reviewed journals.

A list of 19 citations is submitted for participation in the competition – 17 citations in journals indexed and referenced in Scopus/WoS and 2 citations in journals referenced in Zentralblatt Math.

Chief Assistant Iva Dokuzova, PhD was born in 1971 in Sofia. In 1994, she completed her higher education at the Faculty of Mathematics and Informatics with a MSc degree: qualification of a mathematician (major in mathematical structures) and a second qualification of a teacher in mathematics and informatics. From 1996 to 2009, she has worked at the University of Plovdiv, Lyuben Karavelov Branch in Kardzhali. In 2006, she obtained a PhD degree in the scientific specialty *Geometry and Topology* by defending a PhD thesis entitled *On the geometry of almost complex mani-*

folds with Norden metric and Riemannian manifolds with an almost product structure. Since 2009, she holds the position of Chief Assistant at the Department of Algebra and Geometry, Faculty of Mathematics and Informatics, University of Plovdiv. Her teaching experience is more than 28 years.

The scientific interests of Chief Assistant Dokuzova are in the field of differential geometry of smooth manifolds with additional tensor structures (almost complex structure, circulant and skew-circulant structure, almost product structure) and compatible pseudo-Riemannian metrics. She is the author of a total of 29 scientific publications, 12 of which are indexed and referenced in Scopus and/or Web of Science, 1 scientific monograph and 3 textbooks and manuals for students.

Chief Assistant Dokuzova has an extensive teaching experience. In the last 5 years, she has lectured on the following courses: Synthetic geometry, Geometry (Descriptive geometry and Differential geometry), Linear algebra and analytical geometry, Discrete geometric structures, Mathematical foundations of computer graphics (elective course developed by her) to bachelor students and on Random Process Models and Cluster Data Analysis to students from the master program Biostatistics at the Faculty of Mathematics and Informatics. She has taught seminars on 9 disciplines in the same time period. In the last 5 years, she has developed new lecture courses and seminars in 7 academic disciplines. Under her scientific supervision, two students have successfully defended their bachelor or master theses.

The candidate is a member of the Union of Bulgarian Mathematicians and the Geometric society named after Academician Boyan Petkanchin. She is a respected specialist in the geometry field, as evidenced by the list of a total of 21 reviews of scientific manuscripts prepared by her in the last 5 years. She was a member of the Scientific and Organizing Committee of The XVI-th International Conference "Differential Geometry and Dynamical Systems" (DGDS-2022).

Chief Assistant Dr. Dokuzova is a member of the faculty committee on accreditation of FMI. She has participated in the preparation of self-assessment reports for program accreditation, development of new learning plans, a new master program, candidate-student campaigns, etc.

I know Chief Assistant Dr. Iva Dokuzova since 2005. My personal impressions of her are that she is an erudite and accomplished scientist and university teacher; a fair, responsible and valued member of the academic staff of the Faculty of Mathematics and Informatics.

2. General Characteristics of the Candidate's Work
Assessment of educational and pedagogical activity

Chief Assistant Iva Dokuzova, PhD has an extensive teaching experience of over 28 years. The teaching activity report she has submitted shows that in the past five years, her auditorium activity is 1 885 study hours in total, and her overall teaching activity (auditorium and outside the auditorium activity) is 2 426 hours in total. In the last five years, Chief Assistant Dr. Dokuzova is a lecturer in the following obligatory academic disciplines, for which she has developed new lecture courses: Linear algebra and analytical geometry, Synthetic geometry, Geometry (descriptive and differential geometry), Discrete geometric structures, Random Process Models, Cluster Data Analysis and the developed by her undergraduate elective course Mathematical Foundations of Computer Graphics. For participation in the competition Chief Assistant Dr. Dokuzova presented two textbooks for students: "Guide for solving problems in Higher Mathematics" and "Mathematical foundations of computer graphics". The latter textbook is intended for students of the Faculty of Mathematics and Informatics, who study the elective discipline of the same name developed by her. Chief Assistant Dr. Dokuzova is also the author of a manual for solving problems in Linear algebra and analytic geometry for students from bachelor programs in professional direction 4.6. Informatics and Computer Science, published in 2023, which is not included in the materials for participation in the competition. Chief Assistant Dr. Dokuzova was the supervisor of two graduates who successfully defended their theses under her supervision – a bachelor and a master.

My assessment of the educational activity of Chief Assistant Dr. Dokuzova is **positive.**

Assessment of the candidate's scientific and scientific-applied activity

The scientific research of Chief Assistant Iva Dokuzova, PhD are aimed at modern and dynamically developing areas of differential geometry – the study of smooth Riemannian and pseudo-Riemannian manifolds equipped with additional tensor structures of type (1,1): almost product structure, almost complex structure, circulant and skew-circulant structure. For participation in the competition, Chief Assistant Dokuzova has presented 16 of her scientific works, incl. 15 publications in peer-reviewed journals and conference proceedings (9 independent and 6 co-authored – 2 with one co-author and 4 with two co-authors), as well as 1 independent monograph in English. Of the presented papers, 11 are printed in peer-reviewed periodical journals and 4 are published in peer-reviewed volumes of scientific forums – 1 international and 3 national conferences (the annual conferences of the Union of Mathematicians in Bulgaria). According to scientometric indicators, the papers for participation in the competition are distributed as follows:

- 3 papers are in journals with IF – 1 paper with Q2 and 2 papers with Q3;
- 3 papers are in journals with SJR;
- 1 paper is in a journal referenced and indexed in Web of Science;
- 3 papers are in journals referenced and indexed in Zentralblatt and/or MathSciNet;
- 5 papers are in peer-reviewed journals or conference proceedings.

The presented independent monograph of 104 pages, entitled *Four-dimensional Riemannian manifolds with circulant structures and skew-circulant structures*, is reviewed by two reviewers: Assoc. Prof. M. Teofilova and Assoc. Prof. D. Gribacheva.

The documents presented by Chief Assistant Dr. Dokuzova for fulfilling the minimum national requirements and for fulfilling the additional faculty requirements of the Faculty of Mathematics and Informatics for occupying the academic position of associate professor in professional direction 4.5. Mathematics, are correctly written. It is clearly evident from them that with the scientific production presented, the candidate over-fulfills most of the indicators.

Group of Indicators	Candidate's Points	Minimum Points required according to the Regulations for the Development of the Academic Staff in Bulgaria
Group of Indicators A	50	50
Group of Indicators C	100	100
Group of Indicators D	312	200
Group of Indicators E	144	50
Group of Indicators F	80	0

The additional faculty requirements of the Faculty of Mathematics and Informatics for occupying the academic position associate professor in professional direction 4.5. Mathematics are over-fulfilled as follows:

- with a requirement for at least 8 publications, which have not been submitted for the PhD degree, the candidate presents 15 publications and 1 monograph;
- with a requirement for at least 5 publications in periodical journals, the candidate presents 11 publications in periodicals;
- with a requirement for at least 3 publications in journals with IF, the candidate presents 3 publications in journals with IF and 1 monograph in English language, which is equivalent to 2 publications with IF (according to the additional faculty requirements FMI);
- with a requirement of at least 1 textbook or 1 manual, the candidate presents 2 textbooks;
- with a requirement of at least 5 citations, the candidate presents 19 citations in publications (a total list of 41 citations after the defense of the PhD thesis, without self-citations, is also presented).

Dr. Dokuzova has presented a document verifying her participation in 3 research and educational projects and a list of 17 reports at scientific conferences, of which 10 are international forums.

Contributions and citations

The main contributions of Dr. Iva Dokuzova in the scientific works submitted for participation in the competition can be summarized in the following directions:

Introduction and study of new symmetric nonmetric linear connections on classes of Riemannian manifolds with almost product structure and almost complex manifolds with Norden metric – papers [1], [2] and [3].

- An identity for the covariant derivative of the almost product, resp. almost complex structure with respect to the defined connection is proved. This identity is analogous to the characteristic condition for the covariant derivative of the same structure with respect to the Levi-Civita connection by which the corresponding class of manifolds is determined.
- Necessary and sufficient conditions are obtained for the new connections to be equiaffine. The curvature properties of the manifolds in the case when the defined connection is flat are investigated.

Study of the geometry of three-dimensional Riemannian manifolds, for which the metric tensor field and the additional tensor structure are defined by circulant matrices and the third power of the structure is the identity – papers [4], [5], [6], [10], [12], [13] and [15].

- In works [4] and [5], an associated metric is defined, and the almost conformal transformation of the metric is studied. Necessary and sufficient conditions for the circulant structure to be parallel with respect to the conformal connection are obtained. In [5], a particular case of an almost conformal transformation on a manifold with a parallel circulant structure regarding the Levi-Civita connection is studied. The explicit form of the curvature tensor in the case of a locally flat conformal connection is obtained.
- The work [6] is fundamental for the study of three-dimensional Riemannian manifolds with a parallel circulant structure with respect to the Riemannian connection, i.e. an analogue of Kähler manifolds. The existence of an adapted basis as well as an orthogonal basis with respect to such a structure is proved. Sectional curvatures of special tangent planes are studied.
- The work [10] continues the study of basic classes of Riemann circulant manifolds. After the analogue of the class of the Kähler manifolds, in this paper, the author introduces two more classes which are analogues to classes of almost Hermitian manifolds defined by A. Gray and L. Hervella. These classes, denoted L_1 and L_2, are defined by conditions of invariance of the curvature tensor of type (0, 4) with respect to the circulant structure. An associated metric of the Riemannian metric is defined, and by the help of this metric, almost Einstein three-dimensional Riemannian manifolds with a circulant structure are defined. In the same work, the author also studies the relation of almost Einstein manifolds with the manifolds of the two classes mentioned above. Explicit examples of manifolds belonging to the three studied classes are constructed.
- The study of the associated metric is continued in the paper [13], where by analogy with Riemannian manifolds equipped with an almost product structure and almost complex manifolds with Norden metric, a fundamental tensor of type (0,3) and associated 1-forms are defined by help of the covariant derivative of the associated metric with respect to the Levi-Civita connection. A characteristic identity for the fundamental tensor is obtained, analogous to the condition of the main class W_1 of complex manifolds with Norden metric. Relations between the curvature tensors, Ricci tensors and the scalar curvatures of the Riemannian metric and its associated metric are obtained. Examples of Einstein manifolds on Lie groups are constructed and studied.
- In [15], the study of the associated metric and the corresponding associated manifold are continued. Necessary and sufficient conditions for to the associated manifold to belong to the classes L_1, L_2 or the class with a parallel circulant structure regarding the Levi-Civita con-

nection of the associated metric are obtained. Examples on Lie groups are constructed and studied.

Study of three-dimensional Riemannian manifolds for which the metric tensor field and the additional tensor structure are defined by circulant matrices and the third power of the structure is equal to the minus identity – article [8]. The components of the curvature tensor are obtained. A necessary and sufficient condition for the structure to be parallel with respect to Riemannian connection is also obtained. The sectional curvatures of such manifolds are studied and an explicit example of such a manifold is given.

Study of the geometry of four-dimensional Riemannian manifolds for which the metric tensor field and the additional tensor structure are defined by circulant matrices and the fourth power of the structure is the identity – papers [7], [9], [11] and [14], which are included in the first part of the monograph.

- An associated metric is introduced and the almost conformal transformation of the metric is studied. Almost Einstein manifolds are investigated, and sectional curvatures are characterized.

- One of the main contributions of the first part of the monograph is the definition of a Riemannian manifold with an almost product structure associated with an arbitrary four-dimensional Riemannian manifold with a circulant structure. The almost product structure is determined by the second power of the circulant structure and its trace is equal to zero. For such Riemannian manifolds with an almost product structure, the classification of M. Staykova and K. Gribachev is known, which is analogous to the classification of almost complex manifolds with Norden metric by G. Ganchev and A. Borisov. It is proved that the associated manifold is integrable and necessary and sufficient conditions are obtained for it to belong to the main integrable classes W_0, W_1 and W_2, as well as to some classes of the A. Naveira's classification. A number of examples have been constructed and studied.

Study of the geometry of four-dimensional Riemannian manifolds with a skew-circulant structure, where the joint action of the metric and the structure is an isometry – the second part of the monograph.

- An associated metric of the Riemannian metric tensor field is introduced, and a fundamental tensor is defined by its covariant derivative. A characteristic condition for this tensor is obtained. Relations between the curvature tensors and the Ricci tensors for the metric and its associated metric are derived.

- For an arbitrary four-dimensional Riemannian manifold with a skew-circulant structure, an associated almost Hermitian manifold is defined. The almost complex structure is defined by the second degree of the skew-circulant structure. It is established that the associated manifold belongs to the class of locally conformal Kähler manifolds from the Gray and Hervella classification. A necessary and sufficient condition for the skew-circulant structure to be parallel with respect to the Levi-Civita connection is obtained. Examples on Lie groups are constructed and studied.

It can be seen from the above that the results obtained by Dr. Dokuzova significantly enrich and develop the geometry of manifolds with additional tensor structures and compatible Riemannian or pseudo-Riemannian metrics. Dr. Dokuzova's research on Riemannian manifolds equipped with a circulant or a skew-circulant structure shows the close relationship of these two types of manifolds to Riemannian manifolds with a product structure and Hermitian manifolds, respectively.

The declaration of originality and authenticity presented by the candidate is correct.

The personal contribution of Chief Assistant Dr. Dokuzova is undeniable and clearly evident from the fact that she is the independent author of 9 of the publications submitted for participation in the competition and also of the presented monograph.

The scientific works of Chief Assistant Dr. Dokuzova and the results obtained in them have found recognition in the scientific community. This is evidenced by her report of 19 citations for participation in the competition, of which 5 are in publications with IF, 5 are in publications with SJR, 7 are in publications referenced and indexed in Scopus, and 2 are in publications, referenced and indexed in Zentralblatt. Fourteen of these 19 citations are of papers for participation in the competition. A report of a total of 41 post-dissertation citations is also presented, of which 19 are in publications referenced and indexed in WoS, Scopus, Zentralblatt or MatSciNet.

My assessment of the scientific activity of Chief Assistant Dr. Dokuzova is **positive.**

3. Critical Remarks and Suggestions
I do not have any remarks or suggestions to Chief Assistant Iva Dokuzova, PhD.

CONCLUSION

The achieved results of Chief Assistant Iva Rumenova Dokuzova, PhD in her teaching and scientific research activities fully comply with the Law on the Development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for the Implementation of the LDASRB and the relevant Regulations of the University of Plovdiv Paisii Hilendarski. The candidate in the competition has presented a significant number of scientific works published after the defense of the PhD thesis. Her works contain original scientific contributions which have received international recognition.

After getting acquainted with the materials and scientific works presented by the candidate in the competition and analyzing their significance and their scientific and scientific-applied contributions, I confidently give a **positive assessment.** I recommend to the Scientific Jury to prepare a report-proposal to the Faculty Council of the Faculty of Mathematics and Informatics at the University of Plovdiv Paisii Hilendarski for **the election of Chief Assistant Iva Rumenova Dokuzova, PhD in the academic position of Associate Professor** at the University of Plovdiv Paisii Hilendarski in: field of higher education 4. *Natural sciences, mathematics and informatics*, professional direction 4.5. *Mathematics* (*Geometry and Topology*).

March 20, 2024   **The Opinion is prepared by**: ....

                                                         Associate Professor Marta Teofilova, PhD