Annotations of the Scientific Works

by Chief Assistant Professor Iva Rumenova Dokuzova, Ph.D.

Algebra and Geometry Department, Faculty of Mathematics and Informatics,

University of Plovdiv ‘’Paisii Hilendarski’’

For an application in the competition for the academic position "Associate Professor"

Field of Higher education 4. Natural sciences, mathematics and informatics

Professional field 4.5. Mathematics (Geometry and Topology)

Fifteen scientific papers, one monography and two textbooks are submitted for application to the competition. Nine papers are single authored, two of the papers have one co-author and four papers have two co-authors. Eleven works are in scientific journals (eight of them are in foreign journals) and four works are in proceedings of scientific conferences. Three of the papers have an impact factor (total IF (JCR):2,276), six papers have an SJR, seven papers are indexed in Web of Science/Scopus, ten papers are indexed in Zentralblatt Math and/or MathSciNet (AMS).

Scientific papers:

1. **I. Dokuzova**, ***Some properties of one connection on spaces with an almost product structure***, Journal of the Technical University at Plovdiv. Fundamental Sciences and Applications, Series A: Pure and Applied Mathematics (2005-2006), vol. 11, 21--27, ISSN: 1310-8271.
   http://www.tu-plovdiv.bg/content/files/JTU.11+_97.pdf

Abstract. In a Riemannian manifold with a metric g and an almost product structure J we introduce an affine connection $\overline{\nabla}$ by using the Levi-Civita connection ∇ of g. In the basic class of an almost product Riemannian manifolds, with a non-integrable structure J, we obtain that the connection $\overline{\nabla}$ is symmetric and satisfies an analogous characteristic condition, which defines the considered class. In this class, if J is parallel with respect to $\overline{\nabla}$, then J is parallel with respect to ∇. We prove that in the class of locally decomposable manifolds ($\nabla J = 0$), if $\overline{\nabla}$ is a locally flat connection, then ∇ is also a locally flat connection.

2. **I. Dokuzova**, ***An ABJ-connection on almost complex manifolds with Norden metric***, Plovdiv University Scientific Works. Mathematics (2007), 35(3), 47 – 55, ISSN:0204-5249, MR3241381, Zbl 1479.53033.
   http://fmi-plovdiv.org/GetResource?id=241

Abstract. In an almost complex manifold with a Norden metric g we define an affine connection $\overline{\nabla}$ using the Riemannian connection ∇ of g and two smooth vector fields. We obtain that in a class of quasi-Kählerian manifolds (AB-manifolds), the connection $\overline{\nabla}$ is symmetric and satisfies an analogous characteristic condition, which defines the class of quasi-Kählerian manifolds. In this class if J is parallel with respect to $\overline{\nabla}$ then J is parallel with respect to ∇. We consider two particular cases of the transformation $\nabla \to \overline{\nabla}$ in a Kählerian manifold with Norden metric. In both cases we prove that if $\overline{\nabla}$ is a locally flat connection, then ∇ is a locally flat connection.

3. **I. Dokuzova**, ***One connection on a locally decomposable Riemannian* space**, Mathematics and Education in Mathematics, Proceeding of the Thirty Seventh Spring Conference of of the Union of the Bulgarian Mathematicians, Borovets, 2008, 128--131, ISSN: 1313-3330. http://www.math.bas.bg/smb/2008_PK/NP_37PK.pdf

Abstract. In a locally decomposable Riemannian manifold M with a product structure J we define an affine connection $\overline{\nabla}$ using the Riemannian connection ∇ of g. We obtain an invariant tensor field with respect to the transformation $\nabla \to \overline{\nabla}$. In this class, if $\overline{\nabla}$ is a locally flat connection, then M is an almost Einstein manifold. We obtain some curvature properties of M.

4. G. Dzhelepov, D. Razpopov, **I. Dokuzova**, ***Almost conformal transformation in a class of Riemannian manifolds***, In: Research and Education in Mathematics, Informatics and their Applications – REMIA 2010, Proceedings of the Anniversary International Conference REMIA. 10-12.12.2010, Plovdiv, 125-128. ISBN:978-954-423-648-9. http://remia2010.fmi-plovdiv.org

Abstract. We consider a 3-dimensional Riemannian manifold M with a metric g and an affinor structure q, whose third power is the identity. The structure q is parallel with respect to the Riemannian connection ∇ of g. The local components of the tensors q and g form circulant matrices. We defne an almost conformal transformation between g and another metric tensor g_1 in M. Using that definition we construct by induction an infinite series of circulant metrics g, g_1, g_2 ….. , which are successively almost conformaly related. We get relations between the measures of the angles, determined by the vectors x and qx, with respect to the metrics g, g_1, g_2 ….

5. **I. Dokuzova**, D. Razpopov, ***On affine connections in a Riemannian manifold with a circulant metric and two circulant affinor* structures**, Mathematics and Education in Mathematics. Proceedings of the Fortieth Jubilee Spring Conference of the Union of the Bulgarian Mathematicians, Borovets, 2011, 176-181, ISSN:1313-3330. http://www.math.bas.bg/smb/2011_PK/tom/index.html

Abstract. We consider a 3-dimensional Riemannian manifold M with a metric g and a tensor structure q, whose third power is the identity. The local components of the tensors q and g form circulant matrices. We suppose that the structure q is parallel with respect to the Riemannian connection ∇ of *g.* In M we define an almost conformal transformation between g and another metric tensor $\bar{g}$ and find relationship between the curvature tensors *R* and $\bar{R}$ of *g* and $\bar{g}$, respectively. If the Riemannian connection $\bar{\nabla}$ of $\bar{g}$ is locally flat then we obtain some curvature properties of M.

6. G. Dzhelepov, **I. Dokuzova**, D. Razpopov, ***On a three dimensional Riemannian manifold with an additional structure***, Plovdiv University Scientific Works. Mathematics (2011), 38(3), 17--27, ISSN: 0204-5249, MR3243094.
http://fmi-plovdiv.org/GetResource?id=1131

Abstract. We consider a 3-dimensional Riemannian manifold M with a metric g and an additional tensor structure q, where $q^3=id$. The local components of q and g form circulant matrices. We find conditions under which the structure q is parallel with respect to the Riemannian connection ∇ of g. In this case we find relations among the sectional curvatures of arbitrary 2-planes $\{x, qx\}$ $\{x, q^2x\}$, $\{qx, q^2x\}$. We prove that there exists an orthogonal basis of type $\{x, qx, q^2x\}$ in every tangent space of M. By using of an orthonormal basis $\{x, qx, q^2x\}$ we express the sectional curvature of an arbitrary 2-plane by the sectional curvature of the 2-plane $\{x, qx\}$.

7. **I. Dokuzova**, ***Almost conformal transformation in a four dimensional Riemannian manifold with an additional structure***, International Electronic Journal of Geometry (2013), 6(2), 9-13. e-ISSN: 1307-5624, MR3125826 , in Web of Science. https://dergipark.org.tr/tr/download/article-file/769300

Abstract. We consider a 4-dimensional Riemannian manifold M with an additional tensor structure q, whose fourth power is the identity. The components of the metric g and the structure q form circulant matrices. In such a manifold M, when the structure q is parallel with respect to the Riemannian connection of g, we study an almost conformal transformation of g. Let g_1 be the metric obtained by an almost conformal transformation of g. We find conditions under which the structure q is parallel with respect to the Riemannian

connection of g_1. Using that definition we construct by induction an infinite series of circulant metrics g, g_1, g_2 ,..., which are successively almost conformaly related. In a tangent space of the manifold we find the angles, determined by vectors ω and $q\omega$ and by vectors ω and $q^2\omega$, with respect to the metrics g, g_1, g_2,…

8. **I. Dokuzova**, ***Riemannian manifolds with two circulant structures***, Journal of Geometry (2014), 105(3), 529 – 538, doi.org/10.1007/s00022-014-218-2, ISSN: 0047-2468, e-ISSN:1420-8997, MR3267558, SJR(2014):0.348, in Web of Science.
http://link.springer.com/article/10.1007/s00022-014-0218-2

Abstract. We consider a 3-dimensional Riemannian manifold equipped with two circulant structures -- a metric g and a structure q. The third power of q is minus the identity and q acts as an isometry with respect to g. We obtain some curvature properties of this manifold, we give an example of such a manifold and find a necessary and sufficient condition for q to be parallel with respect to the Riemannian connection of g.

9. **I. Dokuzova**, ***Curvature properties of 4-dimensional Riemannian manifolds with a circulant structure***, Journal of Geometry (2017), 108(2), 517--527, doi.org/10.1007/s00022-016-0356-9, ISSN: 0047-2468, e-ISSN:1420-8997, MR3667237 , SJR(2017):0.265, in Web of Science.
http://link.springer.com/article/10.1007/s00022-016-0356-9,

Abstract. We consider a 4-dimensional Riemannian manifold M equipped with a circulant structure q, which acts as an isometry with respect to the metric g and the fourth power of q is the identity. We continue investigations on such a manifold (M, g, q), introduced by D. Razpopov. We obtain expressions for the sectional curvatures of 2-planes {u, qu} and {u, q^2u} by the angles $\angle$(u, qu) and $\angle$(u, q^2u) of the basis vectors of the basis {u, qu, q^2u, q^3u} in a tangent space of (M, g, q). We construct an example of such a manifold on a Lie group and find some of its geometric characteristics.

10. **I. Dokuzova**, ***Almost Einstein manifolds with circulant structures***, Journal of the Korean Mathematical Society (2017), 54(5) 1441--1456, doi.org/10.4134/JKMS.j160524, ISSN: 0304-9914, e-ISSN:2234-3008, MR3691932, IF(2017): 0.684(Q3), SJR(2017):0.439, in Web of Science.
http://pdf.medrang.co.kr/kms01/JKMS/54/JKMS-54-5-1441-1456.pdf,

Abstract. We continue our investigations on a 3-dimensional Riemannian manifold M with a circulant metric g and a circulant structure q, satisfying q^3 = id. The structure q is compatible with g such that an isometry is induced in every tangent space of M. We introduce

three classes ($L_0 \subset L_1 \subset L_2$) of such manifolds (M, g, q). Two of them are determined by special properties of the curvature tensor. The third class is composed by manifolds whose structure q is parallel with respect to the Levi-Civita connection of g. We obtain some curvature properties of these manifolds (M, g, q) and give some explicit examples of manifolds (M, g, q) in L_0, L_1 and L_2.

11. **I. Dokuzova**, D. Razpopov, M. Manev, ***Two types of Lie Groups as 4-dimensional Riemannian manifolds with circulant structure***, Mathematics and Education in Mathematics, Proceedings of the Fourty Seventh Spring Conference the Union of the Bulgarian Mathematicians, Borovets, (2018), 115-120. ISSN:1313-3330. http://www.math.bas.bg/smb/2018_PK/tom_2018/pdf/115-120.pdf .

Abstract. A 4-dimensional Riemannian manifold equipped with an additional tensor structure of type (1,1), whose fourth power is the identity, is considered. The matrix of this structure in some basis is circulant and the structure acts as an isometry with respect to the metric. Such manifolds are constructed on 4-dimensional real Lie groups with Lie algebras of two remarkable types. Some of their geometric characteristics are obtained.

12. **I. Dokuzova**, D. Razpopov, G. Dzhelepov, ***Three-dimensional Riemannian manifolds with circulant structures***, Advances in Mathematics. Scientific Journal (2018), 7(1), 9—16. ISSN: 1857-8365, e-ISSN:1857-8438, ZblMath: Zbl 1432.53020. http://www.research-publication.com/articles/AMSJ/2018/AMSJ-2018-N1-2.pdf

Abstract. We consider a 3-dimensional Riemannian manifold M with two circulant structures – a metric g and an additional structure q with $q^3 = id$. The structure q is compatible with g such that an isometry is induced in any tangent space of M. We calculate the components of the curvature tensor R with respect to the Levi-Civita connection of g. We consider two special properties of R with respect to q and the consequences for some sectional curvatures of this manifold (M, g, q). We give an example of such a manifold.

13. **I. Dokuzova**, ***On a Riemannian manifold with a circulant structure whose third power is the identity***, Filomat (2018), 32(10), 3529-3539. ISSN: 2406-0933, doi.org/10.2298/FIL1810529D. MR301500, IF(2018): 0.789 (Q2), SJR(2018):0.383, in Web of Science.
https://www.pmf.ni.ac.rs/filomat-content/2018/32-10/32-10-12-6661.pdf

Abstract. We study a 3-dimensional Riemannian manifold equipped with an additional tensor structure of type (1, 1), whose third power is the identity. This structure has a circulant matrix with respect to some basis, i.e. the structure is circulant. We define a fundamental

tensor by the metric and by the covariant derivative of the additional structure on such a manifold. This tensor satisfies an important characteristic identity. We establish that the image of the fundamental tensor with respect to the usual conformal transformation satisfies the same identity. We get some curvature properties of the manifold. We construct a Lie group as a manifold of the considered type and find some of its geometrical characteristics.

14. G. Dzhelepov, **I. Dokuzova**, ***Spheres and circles in a tangent space of a 4-dimensional Riemannian manifold with respect to an indefinite metric***, Journal of Geometry (2018), 109:49. ISSN: 0047-2468, e-ISSN:1420-8997, doi.org/10.1007/s00022-018-0455-x, MR3874681, SJR(2018):0.376, in Web of Science. https://link.springer.com/article/10.1007/s00022-018-0455-x

Abstract. Our study is in the tangent space at an arbitrary point on a 4-dimensional Riemannian manifold. This manifold is equipped with an additional tensor structure of type (1, 1), whose fourth power is the identity and the second power is an almost product structure. The metric and the additional structure are compatible, such that an isometry is induced in every tangent space. They determine an associated metric, which is necessarily indefinite. We study spheres and circles, which are given with respect to the associated metric, in some special subspaces of a single tangent space of the manifold.

15. I. Dokuzova, ***On 3-dimensional almost Einstein manifolds with circulant structures***, Turkish Journal of Mathematics (2020), 44(4), 1484-1497. ISSN: 1300-0098, e-ISSN:1303-6149, doi.org/10.3906/mat-1904-97. MR4122919, IF(2020): 0.803 (Q3), SJR(2020):0.454, in Web of Science. https://journals.tubitak.gov.tr/cgi/viewcontent.cgi?article=1293&context=math

Abstract. A 3-dimensional Riemannian manifold equipped with an additional tensor structure of type (1, 1), whose third power is the identity, is considered. This structure and the metric have circulant matrices with respect to some basis. An associated manifold, whose metric is expressed by both structures, is studied. Three classes of such manifolds are considered. Two of them are determined by special properties of the curvature tensor of the manifold. The third class is composed by manifolds whose structure is parallel with respect to the Levi-Civita connection of the metric. Some geometric characteristics of these manifolds are obtained. Examples of such manifolds are given.

Monograph:

1. I. Dokuzova, ***Four-dimensional Riemannian manifolds with circulant structures and skew-circulant structures***, Monograph, 2023, Makros -2001, Plovdiv, 104 pp. ISBN: 978-954-561-577-1.

Abstract. This monograph is organized into 12 sections divided into 2 parts, as well as preliminary notes and an introduction. This work brings together some of the author's research, which are made in the period 2013-2023.

The aim is to observe in parallel two types of Riemannian manifolds with circulant and skew-circulant structures, so that they find their place in the study of differential geometry of manifolds with additional structures. It is also interesting to see the similarities and differences between their geometric properties. Both types of manifolds are associated with known manifolds with additional structures -- with almost product Riemannian manifolds and with almost Hermitian manifolds.

In the present work, two classes of 4-dimensional Riemannian manifolds with additional tensor structures of type (1,1) are considered. The first class includes manifolds with structures whose components form a circulant matrix, and the second class includes manifolds with structures whose components form a skew-circulant matrix. The metric and the circulant (skew-circulant) structure are compatible, and it turns out that the matrix of the metric is also circulant (skew-circulant). The fourth power of the circulant (skew-circulant) additional structure is the identity (minus the identity). In both different cases, the manifolds are equipped with relevant associated metrics, which are necessarily indefinite. Due to the nature of the structures, the manifolds with circulant structures are associated with Riemannian product manifolds, and the manifolds with skew-circulant structures are associated with Hermitian manifolds. The geometrical properties of the considered manifolds are studied.

12. 01. 2024 г.

Plovdiv

Composed by:

Ch. Ass. Prof. Iva Dokuzova