

REVIEW

**from Professor Vasil G. Angelov, DSc.,
Department of Mathematics and Informatics,
University of Mining and Geology "St.Ivan Rilski"**

**on a dissertation work for the acquisition of the educational and academic
degree "Doctor"
at Plovdiv University "Paisiy Hilendarski"**

**Field of higher education 4. Natural Sciences, Mathematics and Informatics,
Professional field 4.5 Mathematics;
Doctoral program -Mathematical Analysis**

Author: Plamena Ivanova Marcheva

Subject: Fixed points and convergence of iterative methods for simultaneous approximation of zeros of polynomials.

Supervisors:

1.Professor Petko Dimitrov Proinov, DSc.

**Department of Mathematical analysis, Faculty of Mathematics and Informatics,
Plovdiv University "P. Hilendarski";**

2.Assoc. Professor Stoil Ivanov Ivanov, PhD.

**Department of Educational Technologies, Faculty of Physics and Technology,
Plovdiv University "P. Hilendarski".**

1. General presentation of the received materials.

By order № P-21-431 of 23.02.2023 of the Rector of Plovdiv University "Paisii Hilendarski" (PU) I was appointed a member of the scientific jury for the earning of the degree Doctor of Philosophy in PU in the field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5 Mathematics, Mathematical Analysis, announced for the needs of the Faculty of Mathematics and Informatics (FMI). The title of the Thesis is, „**Fixed points and convergence of iterative methods for simultaneous approximation of zeros of polynomials**“.

The author of the dissertation is Plamena Ivanova Marcheva - a full-time PhD student at the Department of "Mathematical Analysis" with scientific supervisors Professor Ph.D. Petko Dimitrov Proynov and associate professor Dr. Stoil Ivanov Ivanov from Plovdiv University "Paisiy Hilendarski".

The set of paper materials presented by Plamena Marcheva is in accordance with Art. 36 (1) of the Regulations for the Development of the Academic Staff of the PU and includes the following documents:

- Application to the Rector of the PU to open the procedure for the defense of a dissertation work;
- Curriculum vitae in European format;
- - protocol No. 2 of 17.02.2023 (transcript-excerpt) of an extended CS related to a preliminary discussion of the dissertation work in the "Mathematical Analysis" department;
- - A dissertation with a declaration of originality;
- - A list of scientific publications on the subject of the dissertation;
- Copies of scientific publications;
- - a reference to the requirements of NACID;
- - list of citations;
- - opinion of the scientific supervisors on readiness for defense from 17.01.2023.

The PhD student has attached 3 publications.

2. Brief biographical data

Plamena Marcheva was born on 04/04/1985. She graduated from the "Bachelor's College of Engineering" "Engineering Physics" at the PU "Paisii Hilendarski" in 2013-2017. In 2015-2016 she received the qualification of "mathematics teacher". After that, he received a Master's degree in "Applied Mathematics" in 2017-2018, again at the "Paisii Hilendarski" PU.

From 2017 to 2019, he was a teacher at "Chernorizets Hrabar" Secondary School, Plovdiv. From 01.09.2022, she was appointed as an assistant at the Faculty of Physics and Technology of "Paisii Hilendarski" PU.

By order of the Rector of "Paisiy Hilendarski" PU No. R-33 - 787/ 12.02.2019

Plamena Marcheva is enrolled as a full-time doctoral student in the Field of higher education

4. Natural sciences, mathematics and informatics, professional direction 4.5. Mathematics, doctoral program Mathematical Analysis at the "Mathematical Analysis" department of the Faculty of Mathematics and Informatics (FMI) at Plovdiv University "Paisii Hi-Lendarski" with a duration

of study of 3 years. Prof. Petko Dimitrov Proynov and Assoc. Prof. Stoil Ivanov Ivanov have been appointed as scientific supervisors with the topic of the dissertation "Fixed points and convergence of iterative methods of simultaneous approximation of zeros of polynomials". She was dismissed with the right of defense on 01.03.2022.

3. Relevance of the topic and appropriateness of the set goals and problems

In recent decades, the number of results on simultaneous approximation of the zeros of polynomials, in particular, related to the study of the convergence of iteration methods, has been growing. One of the most important applications of iteration methods is the numerical solution of polynomial equations with coefficients in an arbitrary normed field and the numerical solution of operator equations in Banach spaces. These results are based on the theory of fixed points in metric spaces, and therefore the PhD student has done a historical review of results in this area. The results obtained in the dissertation are a natural continuation of those of Petko Proynov and Stoil Ivanov. In the present dissertation, the convergence of one of the modified Weierstrass methods and of a newly constructed family of simultaneous methods of the Dochev-Burnev type with accelerated convergence is investigated, and the Dochev-Burnev method is generalized.

The relevance of the results obtained in the dissertation work is also confirmed by the published articles on the subject in authoritative scientific journals such as the mathematical journal "Symmetry" from Q2 with an impact factor IF 2.713, as well as AIP Conference Proceedings SJR=0.177.

The PhD student is presented a list of three citations.

4. Knowing the problem

- I think that the doctoral student is well acquainted with the current state and historical development of the examined scientific problems. This statement is based on the following facts:
- – The author's comprehensive historical overview in the introduction to the dissertation work;
- – Content-rich introduction to the dissertation topic. Basic definitions and results of the functional analysis are presented.
- – The References, which includes 108 titles, includes the main results of well-known authors in prestigious scientific journals.

5. Research methodology

The PhD student uses the methods of fixed point theory and convergence methods of iteration methods to simultaneously finding zeros of polynomials. The doctoral student's supervisors made a significant contribution to the development of these methods.

6. Characterization and evaluation of the dissertation work

The current dissertation is dedicated to the study of several methods for simultaneous approximation of zeros of polynomials, with the doctoral student formulating the following main problems:

Problem 1. To investigate the convergence of a modified Weierstrass method and to obtain theorems for local and semi-local convergence that generalize and improve previous results in this direction.

Problem 2. To investigate the local convergence of the Dochev-Burnev method and to obtain theorems that generalize and improve the previous results of this kind.

Problem 3. To construct a family of simultaneous methods of the Dochev-Burnev type with correction. To obtain local and semi-local convergence theorems of the newly constructed family.

Problem 4. To make a numerical implementation of all the studied methods, applying the obtained semi-local convergence theorems for computer verification of their convergence.

The formulated problems are completed within the framework of the dissertation, which contains an Introduction and four chapters.

The *Introduction* of the dissertation contains a comprehensive historical overview, with an emphasis on the topicality of the topic and the scope of the dissertation - the structure of the dissertation and a brief content of the individual chapters and paragraphs.

Chapter 1. General theory of convergence of iterative processes in conically normed spaces

This chapter is of an introductory nature and is devoted to the general theory. Here is an overview of the notations used in the dissertation, as well as the results of the functional analysis necessary for the presentation. The general theory of convergence of iterative processes in conically normalized spaces is presented based on previous results of P. Proynov. The chapter contains 4 Sections.

The main contributions of the thesis are contained in the following Chapters 2 - 4.

Chapter 2. New results for a modified Weierstrass method

Chapter 2 consists of five Sections and is devoted to investigating the convergence of a modified Weierstrass method.

In Section 2.1, the local convergence of the first type (three types are defined depending on the type of function E) of the modified Weierstrass method is investigated. The first main result in the dissertation is also presented here, which improves and complements previous results of the type for this method. This is done in Theorem 2.1.

Section 2.2 investigates the local convergence of the second type of the modified Weierstrass method. The main result in this paragraph is Theorem 2.2, which improves a result of Nejibov [39].

In Section 2.3, a semilocal convergence theorem was obtained - this is exactly Theorem 2.3.

Numerical examples are given in Section 2.4 that show the applicability of Theorem 2.3.

In Section 2.5, the classic and the modified Weierstrass method are compared.

Chapter 3. New results for the Dochev-Burnev method

The third chapter is dedicated to the study of the local convergence of the Dochev-Burnev method. It consists of two Sections.

In Section 3.1, a local convergence theorem of the first type is proved - Theorem 3.1.

In Section 3.2, the second main result in the dissertation is proved - a local convergence theorem of the second type for the Dochev-Burnev method.

Chapter 4. A new family of methods with accelerated convergence

The fourth chapter consists of six paragraphs and is devoted to the study of the convergence of a newly constructed family of simultaneous methods of the Dochev-Burnev type with sharp-root convergence. It contains the third and fourth main results in the dissertation.

In Section 4.1, a new family of simultaneous methods is constructed and its local convergence of the first type is investigated.

In Section 4.2, a local convergence theorem of this family of the second type is proved.

In Section 4.3, as consequences of Theorem 4.1 and Theorem 4.2, local convergence theorems of the first and second type are obtained.

In Section 4.4, a theorem for semilocal convergence of the method of Dochev and Burnev with a correction is proved.

In Section 4.5, as a consequence of Theorem 4.3, semilocal convergence theorems of specific members of the new family are obtained.

In Section 4.6, two numerical examples are discussed that show the applicability of Theorem 4.3.

7. Contributions and significance of the development for science and practice

Basically, I accept the doctoral student's claims for the contributions in the dissertation, namely:

1. Two new local convergence theorems with a priori and a posteriori error estimates and an estimate of the asymptotic constant of a modified Weierstrass method are obtained. The resulting theorems improve and complement previous results of this kind for this method.

2. A semilocal convergence theorem is obtained for the modified Weierstrass method (Theorem 2.3), which improves and complements known results of this kind. This theorem also provides an estimate of the error. Numerical examples are given in which the semilocal convergence theorem is applied to computer check the quadratic convergence of the two methods (Section 2.4).

3. Two theorems for local convergence of the Dochev-Burnev method (Theorem 3.1 and Theorem 3.2) have been obtained, under two different types of initial conditions, with estimates of the error, as well as an estimate of the asymptotic constant. The first of these summarizes, improves and complements previous results of this kind.

4. A family of simultaneous methods of the Dochev-Burnev type (Dochev and Burnev method with correction) with accelerated convergence was constructed.

5. Two local convergence theorems of the Dochev-Burnev method with a correction have been obtained (Theorem 4.1 and Theorem 4.2), as well as local convergence theorems of the first and second kind for four specific methods obtained by using Newton's iteration functions, Weierstrass, Ehrlich and Halley. It is proved that the order of convergence of these methods is four and five.

6. A semi-local convergence theorem is obtained for the Dochev-Burnev method with correction (Theorem 4.3), as well as for the mentioned four specific methods. Numerical examples are given in which these results are applied to computer-verify the convergence of the four methods.

8. Assessment of dissertation publications

There are 3 articles of the doctoral student related to the dissertation. Two articles were published in an international journal with a high impact factor and one article was published in a journal with an impact rank.

Some of the results have been reported at conferences and there is one participation in a contract.

9. Personal participation of the doctoral student

The authorship of the doctoral student in the presented works is indisputable, since they are written in the same manner of presentation. Contribution in co-authored publications is appreciable. Since the law requires an explicit statement of the absence of plagiarism, I declare that I cannot imagine an internationally renowned scientist like Petko Proynov and his student Stoil Ivanov allowing their doctoral student to plagiarize.

10. Abstract

The abstract contains 32 pages and correctly reflects the results obtained, highlighting the main contributions.

11. Critical remarks and recommendations

No mention is made of the papers by Furi, Furi-Vignoli, Weissinger and Matkowski as non-linear generalizations of the Banach-Cacciopoli principle. Their conditions are equivalent to those of F. Browder and Boyd-Wong.

12. Personal impressions

I have no personal impressions of the doctoral student.

13. Recommendations for future use of dissertation contributions and results

I recommend Plamena Marcheva to continue studying with this topic.

CONCLUSION

The dissertation contains new scientific results that comply with the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB), the Regulations for the Implementation of ZRASRB and the relevant Regulations of PU "Paisiy Hilendarski". The presented materials and dissertation results correspond to the specific requirements of the Faculty of Mathematics and Informatics, adopted in connection with the Regulations of the PU for the application of ZRASRB.

The results in the dissertation show that the doctoral student Plamena Ivanova Marcheva possesses theoretical knowledge and professional skills in the scientific specialty Mathematical Analysis, demonstrating qualities and skills for independent conduct of scientific research.

Therefore, I give my **positive assessment of the obtained research**, presented by the above-reviewed dissertation work, abstract, achieved results and contributions, and I recommend to the scientific jury to award the educational and scientific degree "**doctor**"(PhD) to Plamena Ivanova

Marcheva in the field of higher education: 4. Natural sciences, mathematics and informatics, professional direction 4.5 Mathematics, doctoral program Mathematical analysis.

12.03.2023 г.

Referee:

/Professor Vasil G. Angelov/