STATEMENT

by DSc Angel Borisov Dishliev, professor at UCTM-Sofia

on PhD thesis for awarding the educational and scientific degree **„PhD“**

Higher Educational Field: 4. Natural sciences, mathematics and informatics;
Professional Field: 4.5. Mathematics;
Doctoral: Mathematical analysis;
Author of PhD thesis: Plamena Ivanova Marcheva, full-time PhD student at the department "Mathematical Analysis", Faculty of Mathematics and Informatics (FMI), University of Plovdiv Paisii Hilendarski (PU);
Title: Fixed points and convergence of iteration methods for simultaneous approximation of polynomial zeros;
Scientific supervisors: Prof. DSc Petko D. Proinov (PU, Assoc. Prof. Dr Stoil I. Ivanov (PU)

For this review, I will use the relevant instructions of the PU

1. General presentation of the procedure and PhD student

In 2017, Plamena Marcheva graduated bachelor's degree acquiring the qualification of engineer-physicist at Plovdiv University "Paisii Hilendarski". Immediately after that, she continued her education in FMI (PU), and successfully graduated her master degree in mathematics in 2018. In the same year, she was enrolled as a full-time doctoral student at the Department of Mathematical Analysis of PU, PhD program Mathematical Analysis. She was dismissed with the right of defense on 01/03/ 2022. From 01.09. 2022 up to now, P. Marcheva is an assistant at FMI of Plovdiv University. She is an author of 5 scientific publications**.** In the procedure of acquiring the educational and scientific degree "Doctor", she presents three articles. She participates in one scientific project. Her research interests are in the area of mathematical analysis, functional analysis and numerical methods related to the approximate finding the polynomials’ zeros.

The documents presented for obtaining the PhD degree are numerous (so why I omit them). I will note that they meet the specific requirements of the PU.

2. Relevance of the problem developed in the PhD thesis

A number of theoretical and applied mathematical methods are reduced to solving classic problems of the type:

- Finding (with specified accuracy) the zeros of polynomials with coefficients in given normalized fields;
- Solving specific operator equations in the Banach spaces.

Modern and with a wide range of applications in solving the classical problems (mentioned above) is the method of fixed points. Using this universal mathematical approach, it is necessary to solve several additional tasks, guaranteeing the expediency of its use:

- Establishing the existence of the fixed points for a suitable modeling operator defined in the corresponding "working" metric spaces;
- Existence of the corresponding iteration series of approximations to the fixed point of the operator, the elements of which remain in the definition area of the studied operator;
- Finding the “appropriate” starting points of the iteration methods. We note that starting the iteration process from any "suitable" initial point is guaranteed to lead to finding a convergent series of approximations to the sought fixed point;
- Determining the rate of convergence and estimating the error;

- Creation of numerical computer implementations of the specific examples, demonstrating the obtained theoretical methods, etc.;
In the PhD thesis, the subject of the present discussion, another successful attempt is made to combine:
- On the one hand, the “working” mathematical apparatus - fixed point methods;
- On the other hand - iterative simultaneous approximation of all zeros of polynomials. The research in the dissertation work can be counted among the multitude of the modern and important scientific researches. I will remind that a number of Bulgarian mathematicians have contributed to the development of these problems. Here, I will note the names of: B. Sendov, L. Iliev, K. Dochev, P. Burnev, D. Pirgov, V. Angelov, A. Andreev, M. Petkov, N. Kyurkchiev, P. Proynov, S. Ivanov, A. Zahariev, B. Zlatanov, H. Kiskinov, S. Cholakov, I. Nikolova, M. Petkova, M. Vasileva and others. Omitted names are due to the limitations of my knowledge and in no way due to lack of respect.

3. Degree of knowledge of the state of the problem

The PhD student is well acquainted with the main results devoted to the researched problems. At the same time, I will note, that the complete and detailed study of scientific achievements related to the fixed points of operators and numerical finding of polynomials zeros is impossible to be carried out within the limited period of study in the doctoral program. The author's knowledge of a number of specific aspects of the issues under consideration is demonstrated by citing, using and analyzing known specific results of other researchers (in the form of definitions and statements). The brief discussions presented convince me of the depth of knowledge of the candidate. I believe that P. Marcheva knows the current state, the degree of development, the main unsolved tasks, the specific difficulties, and other similar problems that arise during the research of the iterative methods for finding zeros of polynomials that are based on fixed point methods. I am convinced that she can set and solve new mathematical problems from the indicated scientific fields. I will note that this is an important requirement for the acquisition of the educational and scientific degree “doctor”. The literature cited and used is "sparingly selected". By “sparingly selected” literature, I understand that all cited scientific works have a specific application in the author's research on the topic of the PhD thesis. Moreover, some of these results are successfully summarized in the dissertation. I did not see literature references given for the “completeness of exposition” reasons.

4. Research methodology

Using the methods of real and functional analysis, P. Marcheva achieves the formulated goals and solves the specific tasks in his thesis. From these methods, I would indicate:
- Appropriate fixed point methods adapted to the tasks to be solved;
- Iterative methods for convergence of series of approximations.

5. Characterization and estimation of PhD thesis, and contributions

The new results achieved by the PhD student on the topic of the dissertation are contained in Chapters 2, 3, and 4. Several known methods for simultaneously iteratively finding all prime zeros of polynomials of arbitrary degree have been investigated, significantly supplemented and modified. The following methods are studied:
- - Method of K. Weierstrass with a recurrence formula of the type:

$$x^{k+1} = x^k - W\left(x^k\right),\ k = 0,1,...,$$

where:

$$x^k=\left(x_1^k,...,x_n^k\right),\quad W\left(x^k\right)=\left(W_1\left(x^k\right),...,W_n\left(x^k\right)\right),\quad W_i\left(x^k\right)=\frac{f\left(x_i^k\right)}{a_0\prod_{j\neq i}\left(x_i^k-x_j^k\right)},\ i=1,2,...,n.$$

Here n is a degree of polynomial f, and a_0 is a leading coefficient;

- Modified method of K. Weierstrass with a recurrence formula of the form:

$$x^{k+1}=T\left(x^k\right)=\left(\frac{\left(x_1^k\right)^2}{x_1^k+W_1\left(x^k\right)},...,\frac{\left(x_n^k\right)^2}{x_n^k+W_n\left(x^k\right)}\right),\ k=0,1,...;$$

- Method of K. Dochev and P. Burnev (in the form of M. Preshic) with a recurrent formula:

$$x^{k+1}=x^k-D\left(x^k\right),\ k=0,1,...,$$

where:

$$D\left(x^k\right)=\left(D_1\left(x_1^k\right),...,D_n\left(x_n^k\right)\right),\quad D_i\left(x^k\right)=W_i\left(x^k\right)\left(1-\sum_{j\neq i}\frac{W_j\left(x^k\right)}{x_i^k-x_j^k}\right),\ i=1,2,...,n;$$

- A family methods of K. Dochev and P. Burnev with corrections of the form:

$$x^{k+1}=F\left(x^k\right),\ k=0,1,...,$$

where:

$$F\left(x^k\right)=\left(F_1\left(x^k\right),...,F_n\left(x^k\right)\right),$$

$$F_i\left(x^k\right)=x_i^k-2V_i\left(x^k\right)+\left(V_i\left(x^k\right)\right)^2\left(\frac{f'\left(x_i^k\right)}{f\left(x_i^k\right)}-\sum_{j\neq i}\frac{1}{x_i^k-\Omega_j\left(x^k\right)}\right),\ i=1,2,...,n,$$

$$V_i\left(x^k\right)=\frac{f\left(x_i^k\right)}{a_0\prod_{j\neq i}\left(x_i^k-\Omega_j\left(x^k\right)\right)}.$$

Here $\Omega=\left(\Omega_1,...,\Omega_n\right):K^n\to K^n$ is a given iteration function (the family of iteration method depends on this function). Specific cases are considered:

$$\Omega_i\left(x\right)=x_i;\ \Omega_i\left(x\right)=W_i\left(x\right);\quad \Omega_i\left(x\right)=x_i-\frac{f\left(x_i\right)}{f'\left(x_i\right)};\quad \Omega_i\left(x\right)=x_i-\frac{f\left(x_i\right)}{f'\left(x_i\right)-f\left(x_i\right).\sum_{j\neq i}\frac{1}{x_i-x_j}},$$

etc.

In my opinion, the main merits of the presented research are:

- Adaptation of modern and comprehensive methods of the fixed points (methods of Prof. P. Proynov and students) to the set problems for the simultaneous approximation of the simple zeros of polynomials;
- Obtaining new (modified) iteration procedures for simultaneously finding the zeros of polynomials;
- Generalization of known results of other authors in this direction;
- Finding the constraints (referring to the initial point) guaranteeing local and semi-local convergence of the iteration methods;
- Correctness of the iterative vector series of approximations;
- Assessment of the errors;
- Evaluation of the asymptotic constant of the iteration series;
- Numerical implementation (using software packages) of a part of the iteration methods.

6. Evaluation of the publications and personal contribution of the PhD student

On the topic of the doctoral dissertation, the candidate has published three scientific articles. I will make the following classification:

- *In relation to the content of the publications:* The publications are directly related to the dissertation. They are the basis of scientific achievements in it.
- *In connection with the participation of the PhD student:* The three publications have two authors (one of whom is P. Marcheva, and the other is her supervisor). I take the participation of the co-authors as equivalent.
- *In relation to journal rating:* One of the paper is published in a journal with IF=2.713. The article is as follows:

  P. Marcheva, S. Ivanov, Convergence analysis of a modified Weierstrass method for the simultaneous determination of polynomial zeros, Symmetry, 12(9), 2020.

  Another article is published in *AIP Conf. Proc.* (SJR=0.189).
- Citations. The paper with IF cited above is published in 2020, and the other two papers last year. Despite their short period of existence, they are cited three times total. Two of the citations are from foreign authors in the journals with high IF.
- *Fulfillment of minimum national requirements:* They are two:

- Submitted dissertation work-50 points, and this indicator is fulfilled;
- Performance of a group of indicators related to the publication activity and citation of scientific articles of the candidate — at least 30 points. One of the presented paper is published in the journal *Symmetry* with IF, and Q2 quartile. This paper is awarded of 60 points. Another research paper is published in a journal with SJR. The journal is reflected in *Scopus* database. This paper is awarded of 30 points. The PhD thesis results have been cited three times in the journals referred in *Web of Science* and *Scopus*, i.e. 3x8=24 points are awarded. The total number of points achieved by the PhD student in the group of indicators is 114.

7. Abstract

The abstract with the bibliography is placed on 32 standard pages. At the beginning, the relevance of the issues under consideration is substantiated and the objectives of the dissertation are set. It contains a summary of all the main results in the PhD thesis and fully reflects the contributions of Plamena Marcheva. The set goals and specific tasks are clearly formulated. The main concepts and statements in the dissertation are presented respectively in the form of definitions and theorems (without proofs). The applied aspect of theoretical studies is indicated. The conclusion summarizes the main results of the doctoral student. The listed contributions fairly reflect the scientific achievements. A list of the author's publications related to the dissertation is given. It is stated that the achievements of the PhD student have been reported at four different scientific conferences, three of them abroad (North Macedonia, Kosovo and Greece).

The abstract has been prepared according to the requirements. I would add that it is prepared in a form which enables the reader, who is not familiar with the dissertation, to gain a full idea of what is accomplished in it.

8. Recommendations for future usage of contributions and results of dissertation

It is natural that the suggestions for further research of the PhD student will reflect the personal preferences of the recommender. In that sense, I think it's important:

- To "extend" research on the other iterative methods for simultaneously finding the simple zeros of polynomials. This also includes the discovery of other families of methods (similar to the family of methods of K. Dochev – P. Burnev described above), which can

be investigated with the developed technique of the scientific supervisors and the doctoral student;

- Clarifying, supplementing and evaluating some basic and auxiliary functions in the developed methodology. As an example, I will point to the so-called control functions and initial condition functions. The question arises of finding the families of such functions. I believe that, from a practical point of view, appropriate estimates should be obtained for them, which contribute to the easier (albeit rougher) determination of the admissible set of initial points, correct implementation of the iteration procedures and error estimates.

CONCLUSION

The results obtained in the PhD thesis and the comments made above in the opinion give me reason to draw the following conclusions:

1. The dissertation contains theoretical studies in the real and functional analysis, and numerical methods. Research develops and enriches mathematical knowledge. They are an original contribution of the PhD student and are of scientific interest;
2. The statements presented in the dissertation work are useful both for scientists who deal with theoretical problems in the field of fixed points in Banach spaces, and for researchers who apply relevant mathematical methods for approximately finding the zeros of polynomials;
3. The achievements in the PhD thesis meet the requirements of the Law on the Development of the Academic Staff in the Republic of Bulgaria (LDASRB) and the Regulations for the Implementation of LDASRB for the acquisition of the educational and scientific degree "Doctor".
4. I have not established plagiarism.

Due to the facts mentioned above, I rate the research in the dissertation "**positively**".

I propose to the scientific jury to award the educational and scientific degree "doctor" to Plamena Ivanova Marcheva in:

Higher Educational Field: 4. Natural sciences, mathematics and informatics;
Professional Field: 4.5. Mathematics;
Doctoral: Mathematical analysis.

15/ 03/ 2023

Member of the Scientific Jury: .....

Prof. DSc Angel Dishliev