OPINION

by prof. Dr. Sci. Sevdzhan Ahmedov Hakkaev, Shumen University

of a thesis for the award of a scientific degree "Doctor of Science"

research area: 4. Natural sciences, mathematics and informatics

professional field: 4.5. Mathematics (Mathematical Analysis)

Author: Prof. Dr. Boyan Georgiev Zlatanov, University of Plovdiv

Topic: Applications of Coupled Fixed Points and Coupled Best Proximity Points

1. **General presentation of the procedure and the thesis**

By order No. PD-21-1333 dated 18.07.2022 of the Rector of the Plovdiv University "Paisiy Hilendarski" (PU), I have been appointed as a member of the scientific jury to ensure a procedure for the defense of a dissertation " Applications of Coupled Fixed Points and Coupled Best Proximity Points " for the acquisition of the scientific degree "Doctor of Sciences" of the PU in the field of higher education 4. Natural sciences, mathematics and informatics, professional direction 4.5 Mathematics (Mathematical analysis). Author of the dissertation work is Prof. Dr. Boyan Georgiev Zlatanov, Department of Mathematical Analysis at the Faculty of Mathematics and Informatics of the Plovdiv University.

The materials presented by Prof. Dr. Boyan Georgiev Zlatanov is in accordance with Article 45 (4) of the Rules for the Development of the Academic Staff of the Plovdiv University, includes the following documents:

- a request to the Rector of the Plovdiv University to disclose the procedure for the defense of a dissertation work;

– curriculum vitae in European format;

- a copy of the diploma for the educational and scientific degree "doctor";

- protocols from departmental councils related to the opening of the procedure and the preliminary discussion of the dissertation work;

– dissertation work;

– summary;

– a list of scientific publications on the subject of the thesis;

– copies of scientific publications;

- declaration of originality and authenticity of the attached documents;

– certificate of compliance with the minimum national requirements

The applicant has attached 18 publications.

2. **Applicant details**

Prof. Boyan Zlatanov has degree in mathematics form Sofia University. In the period 1997-2001 he was a PhD student in FMI, PU. He get his PhD in 2001. In the period 2001-2008 he was an assistant-professor in Plovdiv University. In the period 2001-2019 he was an associate professor in Plovdiv University. From 2019 he is a professor in Plovdiv University.

3. **Topicality of the subject**

In many areas of mathematics, economics, physics, etc. the existence of solutions of theoretical and applied models is reduced to the existence of fixed points of suitable map. Fixed point theorems, initiated by Banach's contraction principle has proved to be a powerful tool in nonlinear analysis. The theory of fixed points involves the search for combinations of conditions, both for the set X and for the map $T : X \to X$, which ensure that T will leave at least one point of X fixed, i.e. . $\xi = T(\xi)$ for some $\xi \in X$.

The thesis deals with generalizations of Banach's fixed point theorem related to pairs of fixed points and their applications. Generalizations of Ekeland's variational principle, which is related to sets generated by maps with the mixed monotone property, are considered. A

technique is proposed to prove results for the existence of pairs of fixed points for maps with the mixed monotone property using the generalization of the variational principle. Enriched results for existence and uniqueness of pairs of best approximation points with error estimates using series of successive iterations

4. Knowledge of the problem

From the list of cited literature in the dissertation work and the obtained results, it can be seen that the applicant is well familiar with the problems under consideration.

5. Characterization and evaluation of the dissertation work and contributions

In the introduction, the main concepts are defined and previous results related to the goals and problems discussed in the dissertation are presented. Chapter 1 considered the problem of the existence of pairs of fixed points in partially ordered metric spaces that satisfy the mixed monotone property. Separate sections in this chapter are devoted to a generalization of Ekeland's variational principle for maps with the mixed monotone property, fixed point pairs for maps with the mixed monotone property obtained using a variational technique. pairs of fixed points for maps with the mixed Chatterjea -type monotone property obtained using a variational technique. Chapter 2 is devoted to error estimation for pairs of best approximation points. The individual sections of this chapter deal with obtaining error estimates for best approximation points, error estimation for fixed point pairs and best approximation point pairs for cyclic contraction maps, modified fixed point pairs and pairs points of best approximation, existence and uniqueness of pairs of fixed points and points of best approximation for p–cyclic contraction maps.

Chapter 3 is devoted to the problem of a pair of best approximation points in modular function spaces. Points of best approximation for cyclic ρ-contraction maps in modular function spaces, pairs of fixed points and pairs of points of best approximation in modular

function spaces and pairs of fixed points for ρ-Kannan contractions are studied in separate sections in modular function spaces.

Chapter 4 is devoted to the application of fixed-point pairs and best-approximation semi-cyclic maps-pairs in finding market equilibrium in duopoly markets, with separate sections of this chapter devoted to fixed-point pairs of semi-cyclic maps, pairs of fixed points for Hardy–Rogers type semi-cyclic maps, a variational technique in the study of market equilibrium in duopoly markets and their applications.

Chapter 5 is devoted to fixed point triples and best approximation fixed point triples.

6. Assessment of the thesis publications and personal contributions

The thesis consists of an Introduction, 5 chapters and literature from 134 sources in a total of 314 pages. The dissertation is written on the basis of 18 publications, 9 of which are in journals with an impact factor (Q1-4, Q2-5). 6 are in journals indexed in Web of Science and SCOPUS, and the rest are in conferences. 248 of all publications were cited, with 41 citations to dissertation publications.

7. Summary

The auto-reference correctly reflects results and contributions in the dissertation work.

8. **Critical remarks**

I have not critical remarks.

9. Personal impressions

I have known Boyan Zlatanov since 2000 and I highly evaluate his professional activity. His scientific articles present him as a serious researcher.

CONCLUSION

The dissertation contains scientific, scientific-applied and applied results, which represent an original contribution to science (presence/absence of plagiarism) and meet all the requirements of the Ruls of the application of thc Law for the development of the academic

staff in the Republic of Bulgaria (RALDASRB), the Regulations for the Implementation of RALDASRB and the relevant Regulations of Plovdiv University. The presented materials and dissertation results fully correspond to the minimum national requirements, adopted in connection with the Rules of the PU for the application of the RALDASRB.

The dissertation shows that Prof. Dr. Boyan Georgiev Zlatanov possesses in-depth theoretical knowledge and professional skills in the scientific specialty of Mathematical Analysis, demonstrating qualities and skills for conducting research with the receipt of original and significant scientific contributions.

Due to the above, I confidently give my positive assessment of the conducted research, presented by the above-reviewed dissertation work, abstract, achieved results and contributions, and I propose to the honorable scientific jury to award the academic degree "Doctor of Sciences" to Prof. Dr. Boyan Georgiev Zlatanov in the research area 4. Natural sciences, mathematics and informatics, professional field 4.5 Mathematics (Mathematical Analysis).

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Prepared the opinion: ....

(prof. Dr. Sci. Sevdzhan Hakkaev)