OPINION

by Prof. Dr. Hristo Stefanov Kiskinov,

Professor at Plovdiv University "Paisii Hilendarski" (PU),

Faculty of Mathematics and Informatics

of a dissertation for the award of the scientific degree **"Doctor of Science"**
by: area of higher education 4. *Natural Sciences, Mathematics and Informatics*;
professional field 4.5. *Mathematics (Mathematical Analysis).*

Author: Prof. Dr. Boyan Georgiev Zlatanov - PU "Paisiy Hilendarski".
Title: "Applications of Coupled Fixed Points and Coupled Best Proximity Points".

1. General Presentation of the Procedure and the Dissertation

By order No. PD-21-1333 dated 18.07.2022 of the Rector of Plovdiv University "Paisiy Hilendarski" (PU), I have been appointed as a member of the scientific jury to ensure a procedure for the defense of a dissertation work with title " Applications of Coupled Fixed Points and Coupled Best Proximity Points " for the acquisition of the scientific degree "Doctor of Sciences" in the area of higher education 4. *Natural Sciences, Mathematics and Informatics*, professional field 4.5 *Mathematics (Mathematical analysis).* The author of the dissertation is Prof. Dr. Boyan Georgiev Zlatanov, Department of "Mathematical Analysis" at the Faculty of Mathematics and Informatics (FMI) of PU "Paisiy Hilendarski".

The set of paper and electronic materials presented by Prof. Dr. Boyan Zlatanov is in full compliance with Article 45 (4) of the Regulations for the Development of the Academic Staff of the PU (RDASPU) and includes all necessary documents.

The author of the dissertation Boyan Georgiev Zlatanov was born on 29.01.1971 in Plovdiv. In 1991, he graduated from the Plovdiv "Georgi Kirkov" Language High School with intensive study of English. In 1996, he completed the five-year course of study in the specialty Mathematics (second specialty - teacher of mathematics and informatics) at the Faculty of Mathematics and Informatics at the Sofia University "St. Kliment Ohridski" and acquires a Mathematician qualification with a specialization in Mathematical Analysis, equivalent to a Master's degree. In 1997, he was enrolled in doctoral studies at the FMI of the PU. After a competition in 1999, he started working at the FMI of the PU, where he was successively assistant, senior assistant and chief. assistant. In 2001, he defended his PhD-dissertation on "Geometric Properties of Some Classes of Banach Spaces with an Unconditional Basis" by VAK and obtained the educational and scientific degree of Doctor (PhD) of Mathematical Analysis. He obtained his habilitation in 2008, and since 2019 he has held the academic position of professor at the PU. Appears in the NACID register with completed scientometric data. Since 2015, he has been the deputy dean at FMI, responsible for research and project activities under direction 4.5. Mathematics. He was supervisor of a successfully defended doctoral student with a dissertation that fits into the subject of the current dissertation work.

2. Actuality of the Research Topic

The research topic of the dissertation is current, which can be easily established by following the publication activity of those working on this topic in specialized journals.

3. Knowing the Problem

Prof. Dr. Boyan Zlatanov has managed to present a 314-page dissertation that can be read independently. This means that the competent reader, although not a narrow specialist in the researched field, is not obliged to use additional literature when reading. The ability of the author of the dissertation to select the concepts and statements necessary for the exposition, as well as to present his author's contributions in an appropriate way, so that the dissertation is easy and pleasant to read, speak unequivocally of deep knowledge in the researched field.

4. Research Methodology

The research methodology is the standard one for any mathematical research – proving statements, creating models and constructing examples.

5. Characterization and Evaluation of the Dissertation Work and Contributions - Presence/Absence of Plagiarism

The dissertation is devoted to various substantive generalizations of the famous Banach fixed point theorem and their applications. It is written in 314 pages and consists of a preface, an introduction, five chapters, a conclusion and a bibliography of 134 titles. The introduction is of an overview nature and it sets out the main concepts and statements that are used in the following chapters. In the first chapter, a generalization of Ekeland's variational principle for maps with a mixed monotone property and the results obtained with its help for coupled fixed points in partially ordered metric spaces are presented. Chapter two is devoted to error estimation for coupled best proximity points. Chapter three deals with coupled best proximity points in modular function spaces. In the fourth chapter various applications of coupled fixed points and coupled best proximity points of semi-cyclic maps in economics are studied, specifically in the study of equilibrium in duopoly markets. Chapter five summarizes some of the previously described results for triples of fixed points and, respectively, triples of best proximity points. In the conclusion, the author has made a self-assessment of the contributions contained in the dissertation work and has described the approbation of the obtained results.

I fully support the main contributions in this dissertation described by the author. Namely:

- A generalization of Ekeland's variational principle was made and with its help conditions for existence and additional ones for uniqueness of coupled fixed points for classes of maps with mixed monotonic property were found.
- Error estimates for couples and triples of best proximity points are found.
- Results for ordered pairs of best proximity points and for cyclic maps by introducing modified cyclic maps and points are generalized.
- The notion of best proximity points for maps in modular function spaces is generalized.
- A new model is presented to study the existence and uniqueness of equilibrium in duopoly markets using the notion of an ordered pair of semi-cyclic maps introduced by the author.
- Possibilities for generalization of the most of the investigated problems for triple fixed points and triple best proximity points are considered.

An undoubted contribution of the present dissertation is that for all obtained and described results, numerous possible real applications have been specified.

I do not detect "plagiarism" in the works of the author and the presented thesis in the sense of the Law on the Development of the Academic Staff in the Republic of Bulgaria.

6. Assessment of the Dissertation's Publications and Personal Contributions of the Author

The presented dissertation is based on 18 publications (according to the attached list of dissertation's publications), of which 16 are in journals and 2 are in conference proceedings. For participation in the procedure, according to the attached list and forming a total of 618 points with a minimum national requirement of 100 points, a total of 11 articles were published in journals indexed in WoS and SCOPUS (4 with IF-Q1, 4 with IF-Q2 , 2 in SCOPUS with SJR>0 and 1 in WoS and SCOPUS without IF and SJR). Two publications have already been "used" in the competition for professor, they have not been scored and it has been correctly noted that they do not participate in the current procedure. Two publications that are in press (one from AIP already published but not yet indexed) and one that does not carry points are also not on the list of articles to participate in the procedure.

From the 11 articles participating in the procedure, 3 are single-authored, 2 are with 1 co-author, 3 are with 2 co-authors, 2 are with 3 co-authors, and 1 is with 4. For me, the personal

contribution of the candidate in the co-authored articles is undoubted, and by my personal opinion, he is the leading author in them.

The list of noticed citations of the dissertation publications (compiled at the beginning of July) includes 41 titles, of which 38 are in journals indexed in the world-famous databases (Web of Science, Scopus, Zentralblatt Math, Mathematical Reviews). In the current procedure, Prof. Zlatanov participated with 20 citations, unused in other procedures, which form a total of 156 points, which exceeds the minimum national requirement of 100 points. Meanwhile, now, two months after the lists were compiled, it can be noted few new citations. I am extremely impressed by the fact that the authors of the citing publications are a variety of scientists from all over the world (and not from a limited circle of "super specialists" profiled in a very narrow area), which is a sure sign of the importance and relevance of the results obtained by Prof. Zlatanov.

I have no major critical remarks. I can only point out that the table on page 300 of the relations between the contributions, the location in the thesis and the publications is a little oddly constructed, and also that a few typographical errors have crept in, which is, however, fully justified in view of the large volume of dissertation work.

7. Summary

The Summary, written in Bulgarian and in English, has 64 pages, corresponds to the requirements of RDASPU and contains the main results obtained in the dissertation work. All statements are presented without proofs.

8. Recommendations for Future Use of Dissertation Contributions and Results

In each chapter, the author of the dissertation has presented his views on possible future developments on the research area, so I can only wish him to continue his work with the same enthusiasm.

CONCLUSION

The dissertation ***contains scientific, scientific-applied and applied results, which are an original contribution to the science*** and **meet all the requirements** of the Law on the Development of the Academic Staff in the Republic of Bulgaria (LDASRB), the Regulations for the Implementation of the LDASRB and the relevant Regulations of Plovdiv University "Paisii Hilendarski". I detect no plagiarism. The presented materials and dissertation results far exceed the minimum national requirements introduced by the Regulations for the Implementation of the LDASRB.

The dissertation work shows that Prof. Dr. Boyan Georgiev Zlatanov possesses in-depth theoretical knowledge and professional skills in the scientific specialty Mathematical Analysis, **demonstrating** qualities and skills for conducting research with obtaining original and significant scientific contributions.

Due to the above, I confidently give my positive assessment of the conducted research, presented by the above-reviewed dissertation work, summary, achieved results and contributions, and ***I propose to the honorable scientific jury to award the scientific degree "Doctor of Sciences"*** to Prof. Dr. Boyan Georgiev Zlatanov in the area of higher education 4. *Natural sciences, Mathematics and Informatics*; professional field 4.5. *Mathematics (Mathematical Analysis).*

06.09. 2022 г.   Signature: ....

Plovdiv   Prof. Dr. Hristo Stefanov Kiskinov