ANNOTATION OF THE PROVIDED DOCUMENTS AND SELF-ASSESSMENT OF DR. STOIL IVANOV IVANOV

I. Annotation of the provided documents.

1. I have provided compliance reports, which show that I meet all the requirements of the laws of the Republic of Bulgaria and the rules of the Faculty of Mathematics and Informatics, University of Plovdiv Paisii Hilendarski concerning the academic position **`Associate Professor'** in area of higher education *4. Natural Sciences, Mathematics and Computing; 4.5. Mathematics (Mathematical analysis).*
2. On 09 May 2014, I have acquired a PhD degree in the area of higher education *4. Natural Sciences, Mathematics and Computing; 4.5. Mathematics (Mathematical analysis).*
3. Since February 2013, I work as `Assistant Professor' in the Faculty of Physics and in the Faculty of Physics and Technology of University of Plovdiv Paisii Hilendarski.
4. For the **`Associate Professor'** position I apply two publications in scientific journals with impact factor (IF) in exchange of the required habilitation thesis.
5. Apart from the above mentioned papers, I apply five publications in journals with IF, a publication in journal with SJR and two publications in Conference Proceedings.
6. I declare that all of the presented results are original.

II. Self-assessment of the contributions made.

The author's contributions are mainly in the following two directions:

- Study of high-order iteration methods for individual approximation of simple and multiple polynomial zeros;
- Study of high-order iteration methods for simultaneous approximation of simple and multiple polynomial zeros;

Since 2009, Proinov has developed a general convergence theory of the Picard iteration in cone metric spaces and in n-dimensional vector spaces. All results in the presented articles are based on that theory. In **[2]**, we apply Proinov's theory to obtain two local convergence theorems about the Chebyshev's iterative method for the computation of multiple polynomial zeros. The obtained theorems give exact estimations of the convergence domains as well as a priori and a posteriori error estimates right from the first iteration. All of the results are new even in the case of simple zero.

In 1891, Weierstrass established the first iterative method for the simultaneous computation of all zeros of a complex polynomial. Such methods are often called just *simultaneous methods.* It is well known that the *Weierstrass method* is quadratically convergent for simple zeros. In 1964, Dochev and Byrnev presented the first cubically convergent simultaneous method and three years later, Ehrlich introduced and studied another third-order simultaneous method. It is interesting to note that the *Dochev-Byrnev's method* and *Ehrlich's method* are also known in the literature as *Tanabe's method* and *Börsch-Supan's method*, respectively. In **[5]**, we construct a one-parameter family of cubically convergent simultaneous algorithms that includes as special cases the above mentioned two remarkable methods. A semilocal convergence theorem with computable initial conditions and error estimates is established. The best convergence radius is shown to be for the Ehrlich's method.

In 1977, Nourein presented two new fourth-order simultaneous methods. In 1991, the Japanese mathematicians Sakurai, Torii and Sugiura constructed and studied another efficient fourth-order simultaneous method. In **[7], [8]** and **[9]**, we prove several local and semilocal convergence theorems that generalize, improve and complement all existing results about the Sakurai-Torii-Sugiura's method.

In the paper **[10]**, a new fifth-order family of simultaneous methods of Gander's type was constructed and studied. Local and semilocal convergence theorems that provide exact estimations of the convergence domains as well as a priori and a posteriori error estimates right from the first iteration have been presented.

In 2002, Batra has considered the classical Newton's method as a method in n-dimensional vector space, i.e., as a method for finding all the zeros of a complex polynomial simultaneously. Motivated by this Batra's work, in **[1]** we study the Halley's method as a method for simultaneous computation of all zeros of a polynomial. New local and semilocal convergence theorems with a priori and a posteriori error estimates have been proven. As a continuation of the mentioned work, in **[4]** the local convergence of the famous Newton, Halley and Chebyshev's iterative methods considered as methods for simultaneous determination of all multiple zeros of a polynomial has been investigated.

It is well known that the Newton's method is quadratically convergent while the Halley and Chebyshev's methods are cubically convergent to multiple polynomial zeros. However, all these methods demand the prior knowledge of the multiplicities of the zeros which highly reduces their practical applicability. In 1870, Schröder presented a second-order iterative method for approximation of simple and multiple polynomial zeros that does not require the knowledge of the multiplicities of the roots. In **[3]** and **[6]**, the Schröder's method has been investigated as a method for the simultaneous approximation of all simple and multiple polynomial zeros. In **[6]**, a general local convergence theorem of the Picard iteration in n-dimensional vector spaces has been proved and used for obtaining of a local convergence theorem which in turn has been used for the proof of a theorem with computable initial conditions and error estimates about the mentioned Schröder's method which are of significant practical importance.

III. Abstracts of the presented papers.

[1] P.D. Proinov, S.I. Ivanov. On the convergence of Halley's method for simultaneous computation of polynomial zeros. J. Numer. Math., 23(4):379-394, 2015.**Q3(IF0.552).**

In this paper, we study the convergence of the famous Halley's method considered as a method in n-dimensional vector space, i.e., as a method for finding all the zeros of a polynomial simultaneously. Two types of local convergence theorems under different types of initial conditions as well as a convergence theorem under computationally verifiable initial conditions are established. Numerical examples are presented to show the applicability of the semilocal convergence theorem for numerical prove of the cubic convergence of the Halley's method.

[2] S.I. Ivanov. On the convergence of Chebyshev's method for multiple polynomial zeros. Results. Math., 69(1):93–103, 2016. **Q2 (IF 0.693).**

In this paper, the local convergence of Chebyshev's iterative method for the computation of a multiple polynomial zero is investigated. Two convergence theorems for polynomials over an arbitrary valued field are proved. A priori and a posteriori error estimates are also provided. All of the results are new even in the case of simple zero.

[3] V.K. Kyncheva, V.V. Yotov, S.I. Ivanov. A theorem for local convergence of Schröder's method for simultaneous finding polynomial zeros of unknown multiplicity. In: Renewable Energy & Innovative Technologies: Conference Proceedings, Smolyan, Bulgaria, 2016, ISBN 978-619-7180-78-7, pp. 192–193.

In this talk we consider the Schröder's iterative method as a method for the simultaneous finding polynomial zeros of unknown multiplicity. A local convergence theorem with a priori and a posteriori error estimates is stated.

[4] V.K. Kyncheva, V.V. Yotov, S.I. Ivanov. Convergence of Newton, Halley and Chebyshev iterative methods as methods for simultaneous determination of multiple polynomial zeros. Appl. Numer. Math., 112:146-154, 2017. **Q1 (IF 1.263).**

In this paper, we provide a local convergence analysis of Newton, Halley and Chebyshev's iterative methods considered as methods for the simultaneous determination of all multiple zeros of a polynomial over an arbitrary valued field. Convergence theorems with a priori and a posteriori error estimates for each of the proposed methods are established. The obtained results for Newton and Chebyshev's methods are new even in the case of simple zeros. Three numerical examples are given to compare the convergence properties of the considered methods and to confirm the theoretical results.

[5] S.I. Ivanov. A unified semilocal convergence analysis of a family of iterative algorithms for computing all zeros of a polynomial simultaneously. Numer. Algor., 75:1193–1204, 2017. **Q1 (IF 1.536).**

In this paper, we first present a new family of iterative algorithms for the simultaneous determination of all zeros of a polynomial. This family contains two well-known algorithms namely the Dochev-Byrnev's method and the Ehrlich's method. Second, using Proinov's approach to studying convergence of iterative methods for polynomial zeros, we provide a semilocal convergence theorem that unifies the results of Proinov (Appl. Math. Comput. 284: 102–114, 2016) for Dochev-Byrnev and Ehrlich's methods. We present two numerical examples to show some practical applications of the obtained result.

[6] V.K. Kyncheva, V.V. Yotov, S.I. Ivanov. On the convergence of Schröder's method for the simultaneous computation of polynomial zeros of unknown multiplicity. Calcolo, 54(4):1199–1212, 2017. **Q1 (IF 1.603).**

In this paper we make three contributions. First, we establish a general theorem for iteration functions in a cone normed space over an n-dimensional real vector space. Second, using this theorem together with a general convergence theorem of Proinov (J Complex 33:118–144, 2016), we obtain a local convergence theorem with a priori and a posteriori error estimates for the Schröder's iterative method considered as a method for simultaneous computation of all polynomial zeros of unknown multiplicity. Third, using a Proinov's approach, we transform our local convergence theorem to a theorem under computationally verifiable initial conditions. Numerical examples which demonstrate the convergence properties of the proposed method are also provided.

[7] P.D. Proinov, S.I. Ivanov. Semilocal convergence of Sakurai-Torii-Sugiura method for simultaneous approximation of polynomial zeros. In: Y. Simsek, ed., Proceedings Book of MICOPAM2018, Antalya, Turkey, 2018, ISBN 978-86-6016-036-4, pp. 94-98.

In this talk, we provide a new semilocal convergence theorem for a fourth order iterative method for the simultaneous approximation of polynomial zeros due to Sakurai, Torii and Sugiura. This theorem improves and complements the existing result of Petković et. al. Two numerical examples are given to show some practical applications of our result.

[8] P.D. Proinov, S.I. Ivanov. On the local convergence of Sakurai-Torii-Sugiura method for simultaneous approximation of polynomial zeros. AIP Conf. Proc., 2116: Article No. 450027, 3 pages, 2019. **(SJR 0.190).**

In this talk, we discuss on two new results for the convergence of a fourth-order iterative method due to Sakurai, Torii and Sugiura. We provide two local convergence results for this method. The first one improves a result of Petković while the second one is the first such kind of result about this method.

[9] P.D. Proinov, S.I. Ivanov. Convergence analysis of Sakurai-Torii-Sugiura iterative method for simultaneous approximation of polynomial zeros. J. Comput. Appl. Math., 357: 56–70, 2019. **Q1 (IF 2.037).**

In this paper, we provide a detailed local and semilocal convergence analysis of the Sakurai–Torii–Sugiura iteration method. Our results improve and complement all existing convergence results about this method. Computationally verifiable initial conditions which are of significant practical importance as well as a priori and a posteriori error estimates are also provided. Two numerical examples are given to show some practical applications of our semilocal results.

[10] P.D. Proinov, S.I. Ivanov, M.S. Petković. On the convergence of Gander's type family of iterative methods for simultaneous approximation of polynomial zeros. Appl. Math. Comput., 349:168–183, 2019. **Q1 (IF 3.472).**

In this paper, we propose a fifth-order family of iterative methods for approximation of all zeros of a polynomial simultaneously. The new family is developed by combining Gander's third-order family of iterative methods with the second-order Weierstrass root-finding method. The aim of the paper is to state initial conditions that provide local and semilocal convergence of the proposed methods as well as a priori and a posteriori error estimates. In the case of semilocal convergence the initial conditions and error estimates are computationally verifiable which is of practical importance.

Date 25.05.2021 Applicant: ....

Plovdiv Assist. Prof. Dr. Stoil Ivanov