

REVIEW

by

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of the materials submitted for the participation on the competition for the academic position of “Associate Professor” for the needs of Faculty of Physics and Technology of Plovdiv University "Paisii Hilendarski" (PU),

Scientific field: 4. Natural Sciences, Mathematics and Informatics;

Professional field: 4.5. Mathematics;

Scientific specialty: Mathematical analysis

The competition for “Associate Professor” was announced in the State Gazette (SG) , № 40/14.05.2021) and on the website of PU. The academic position is for the needs of the Department of Educational Technologies of Faculty of Physics and Technology. The only candidate for participation in the competition is Ch. Assistant Professor **Dr. Stoil Ivanov Ivanov** from the same department.

For preparing this review I will follow the instructions of PU.

1. GENERAL PRESENTATION OF THE MATERIALS RECEIVED

By order № P33-3130 of 12.07.2021 of the Rector of the Plovdiv University I was appointed a regular member of the scientific jury of the competition for the above academic position. At the first meeting of the Scientific Jury I was chosen to prepare a review of this competition.

The set of electronic materials presented by Dr. S. Ivanov is in accordance with the Regulations for the development of the academic staff of Plovdiv University "Paisii Hilendarski". The set includes the following documents.

1. Application to the Rector of PU for admission to participation in the competition;
2. Curriculum vitae in European format;
3. Diploma for educational qualification degree "Master";
4. Diploma for educational and scientific degree "Doctor";
5. List of scientific publications and university teaching aids;
6. List of citations;
7. Information on compliance with the minimum national requirements;
8. Information on compliance with the additional requirements of FMI;
9. Annotation of the materials and self-assessment of the contributions in Bulgarian and English;
10. Declaration of originality and authenticity of the attached documents;
11. Certificate of work experience;
12. Reference for educational work;
13. Copies of teaching aids and scientific publications for participation in the competition;
14. List of prepared reviews;

15. Official note for participation in projects;
16. List of participations in conferences and seminars.

The candidate Dr. S. Ivanov has submitted for participation in the competition a total of 10 scientific publications and one textbook. I accept for review all submitted materials because:

- They do not been used in the candidate's dissertation;
- They correspond to the field, professional field and the scientific specialty of the discussed competition;
- The results obtained in the presented scientific papers do not match;
- I have no doubt that the results have been misappropriated by other authors (i.e. there is not a plagiarism).

2. BRIEF BIOGRAPHY

The candidate for the academic position of "Associate Professor" completes the following educational qualification degrees (EQD):

№	Period	EQD	Qualification	University
1	1993-1996	Professional Bachelor	Engineer-transport operator	"Todor Kableshkov" University of Transport
2	1997-2002	Bachelor	Teacher of mathematics and computer science	PU "Paisii Hilendarski"
3	2002-2003	Master	Mathematician	PU "Paisii Hilendarski"
4	2007-2010	Master	Engineer	"Todor Kableshkov" University of Transport
5	2010-2014	PhD	Mathematical Analysis	PU "Paisii Hilendarski"

The PhD thesis of Dr. S. Ivanov is: "Convergence of Halley iterative method for individual and simultaneous approximation of polynomial's zeros". The dissertation was defended in 2014.

The professional realization of the candidate is as follows:

№	Period	Position	University
1	2013 - 2015	Assistant	PU "Paisii Hilendarski"
2	2015 - ...	Chief Assistant	PU "Paisii Hilendarski"

The scientific activity of the candidate is determined most objectively through his scientific activity, which consists of a relatively large number of high quality scientific papers (a total of 21 scientific papers). An important factor is their coverage by the scientific community (expressed in a total of 72 citations). The candidate has participated of 18 international and national scientific forums. Dr. S. Ivanov has participated in 1 research

project at the Research Fund and 5 research projects at the Research Fund at PU "Paisii Hilendarski". He has prepared nearly 40 reviews of scientific articles (almost all of them have been proposed for publication in journals with impact factor and impact rank). He is an author of several reviews of student theses at Plovdiv University. The candidate's scientific interests are diverse and include: iterative methods, polynomials zeros, fixed points, differential geometry, etc.

3. MAIN NUMBERS INDICATORS OF THE APPLICANT'S ACTIVITY

3.1. Publications for participation in the competition: The list includes one university manual and 10 scientific publications.

The textbook is a guide for solving problems in linear algebra and analytical geometry. The manual has two co-authors (one of which is the candidate) and is published by the university Publishing house "Paisii Hilendarski".

Three of the publications are in conferences volumes. One of these conferences were held in Bulgaria and two abroad (Turkey and Greece). One of these published papers, has an impact rank, refers to 16th International Conference of Numerical Analysis and Applied Mathematics, Rhodes, Greece. The other seven publications of the candidate, given for the competition, were published in journals with IF and SJR, which are referenced in Web of Science and Scopus. The quality of these scientific articles can be judged in view of high classification of the journals in which they are published, namely:

Number of publications	Classification of the scientific journals	Average impact factor
5	Q1	1,982
1	Q2	0,693
1	Q3	0,552

According to the number of authors, the publications can be divided as follows:

- 2 are with one author;
- 5 have two co-authors;
- 3 have three co-authors

3.2. All publications of the candidate: The list of publications of Dr. S. Ivanov can be divided into four groups (depending on their purpose):

- **First group:** Publications for participation in the competition for "associate professor" (11 scientific papers, described in detail in the previous point);
- **Second group:** Publications for the acquisition of Educational scientific degree "Doctor" (3 scientific papers, two of which have been published in journals with impact factor, referenced in Web of Science and Scopus);
- **Third group:** Publications for participation in the competition for "Chief Assistant" (2 scientific papers, one of which is in a journal with an impact rank without an impact factor);
- **Fourth group:** Other publications (6 scientific papers, three of which have been published in journals with impact factor, referenced in Web of Science and Scopus, two

of these papers are in journals without impact factor and impact rank and one scientific paper is included in volume at a conference abroad).

Summarizing the previous data, we come to the conclusion that the scientific work of the candidate consists of 21 scientific papers:

- 1 is a textbook;
- 12 are in the journals with impact factor, referred to Web of Science and Scopus;
- 2 are in journals or conference volumes with impact rank without impact factor;
- 6 are in journals or conference volumes that are not indexed.

3.3. Citation of the scientific papers of the candidate: 72 citations (without auto-citation) obtained of 14 scientific papers are presented. Quotes can be divided into two groups:

- **First group:** 38 citations in the journals with an impact factor or an impact rank;
- **Second group:** 34 citations in the journals that are not indexed.

3.4. Information on fulfillment of the minimum national requirements, as well as of the minimum requirements of FMI:

The fulfillment of the minimum national requirements for holding the academic position of "associate professor" is shown in the following table:

National indicators	Minimum points	Submitted materials by the applicant	Points achieved
A. PhD thesis	50	PhD thesis	50
B. Habilitation - scientific publications	100	2 publications in the journals with Q1 and Q3 in WoS and Scopus: 1x75 = 75 points; 1x45 = 45 points	120
C. Scientific publications (outside the Habilitation)	200	6 publications in the journals with the following classification Q1 - 4, i.e. 4x75 = 300 points; Q2 - 1, i.e. 1x60 = 60 points; SJR - 1, i.e. 1x30 = 30 points	390
D. Citations in scientific journals	50	There are 38 citations from the publications which are referenced in WoS and Scopus 38x8 = 304 points	304

The table above shows that each of the minimum national indicators is over fulfilled. The last indicator (D. Cited in scientific journals) is exceeded more than 6 times - not all achievements of the candidate are taken into account.

The fulfillment of the minimum requirements of FMI for holding the academic position "Associate Professor" is shown in the following table:

№	Minimum requirements of FMI	Performance
1	8 publications not given for acquisition of Doctor degree and for the academic position "Chief Assistant"	10 publications
2	5 publications in the journals	8 publications
3	3 publications in the journals with IF	7 publications
4	1 manual	1 manual
5	5 citations	72 citations

The minimum requirements of FMI are met by the candidate.

4. GENERAL CHARACTERISTICS OF THE ACTIVITY OF THE CANDIDATE

4.1. Assessment of educational and pedagogical activity of the candidate: The candidate has many years of rich teaching experience gained in PU. Since 2013, he has conducted a number of lecture courses, seminars and exercises in the disciplines:

- Linear algebra and analytic geometry;
- Mathematical analysis, part 1;
- Mathematical analysis, part 2;
- Probability theory and mathematical statistics;
- Mathematical methods of physics;
- Mathematics 1 (Algebra and differential calculus);
- Mathematics 2 (Integral calculus and differential equations);
- Spaces and operators in physics (elective course);
- Applied functional analysis (elective course).

Indeed, serious teaching practice.

He was a reviewer of 4 diploma theses of students from PU. He is the head of a successfully defended bachelor's degree.

The presented manual (184 pages) is intended for the students, mainly of the University of Plovdiv, who study linear algebra and analytical geometry in more details. The manual content is within the generally accepted limits of the academic discipline taught in the bachelor's degree. The structure of each main topic (14 in total) is according to the accepted classical standards:

- summary of the necessary theoretical materials: basic definitions, introducing the concepts and the relations between them; main statements on the topic; important remarks clarifying the meaning of definitions and theorems;
- examples with solutions, through which the theory is made meaningful and the applied character of the discipline is established;
- unsolved examples with answers. Through them, the students consolidate the acquired knowledge and are educated to work independently.

I find the guide useful for students. Personally, I was very impressed by its content and professional layout.

4.2. Assessment of the candidate's scientific activity: In general, the suggested scientific results complete, enrich and summarize the scientific knowledge on certain topics of the theory of iterative methods for individual and simultaneous finding the zeros of polynomials. In essence, its results are formulation and proof of new scientific facts and the creation, and improvement of new iterative methods. The scientific work of S. Ivanov (although partially reduced for participation in the discussed competition) is diverse and largely original in terms of specific topics studied. The results are mostly theoretical in its nature, but are provoked of studying the real problems. In some presented papers, numerical examples are considered, through which the theory is additionally visualized, the achieved results are analyzed, and different iterative procedures are compared. The presented works significantly exceed the requirements (as well as my expectations) for the

quantity and quality of scientific papers for the academic position of "associate professor" in a prestigious university, such as PU.

In article [2] (from the list of the author of the scientific papers for participation in the competition for "Associate professor") an iterative procedure for finding an individual zero of a polynomial $f = f(x) = a_0(x - \xi_1) \dots (x - \xi_n)$ is investigated. The multiplicity of the approximate zero of the polynomial is known in advance. The main working tool is a general theorem of prof. P. Proinov for convergence in metric spaces of iterative procedures of the type of E. Picard:

$$x_{k+1} = Tx_k, \quad k = 0, 1, \dots$$

Under the theorem, conditions of the convergence of the series $\{x_k\}$ is valid regardless of the choice of the initial point x_0 of the domain of operator T . We will note that in most of the considered iterative methods in the papers of this competition, this circumstance is not valid, i.e. the convergence is not global. In addition, the results used by P. Proinov give the order of the error of the members of the convergent series with respect to the boundary point, which is zero of the original polynomial. In the mentioned work (number [2]), the iterative method of P. Chebishev is studied:

$$x_{k+1} = \begin{cases} x_k - \frac{m^2}{2} \left(\frac{3-m}{m} + \frac{f(x_k)f''(x_k)}{(f'(x_k))^2} \right), & f'(x_k) \neq 0; \\ x_k, & f'(x_k) = 0. \end{cases}$$

Here m is the multiplicity of the required zero. The maximum radius of deviation of the initial point x_0 from the unknown zero ξ , at which the iterative method is convergent, is established. The degree of the polynomial, the multiplicity of zero and the minimum distance between the required zero and the other zeros of the polynomial are involved in determining this radius. Interesting error estimates are obtained (i.e. the distance from each element of the iterative sequence to the zero of the polynomial).

The remaining papers in the competition are devoted to the methods for simultaneously finding zeros of polynomials (simultaneous methods). This mathematical theory started from the works of K. Weierstrass at the end of the 19th century in the case when the zeros of the polynomial are simple (single). The recurrent formula introduced by him has the form:

$$x^{k+1} = x^k - W(x^k), \quad k = 0, 1, \dots, \quad (a)$$

where n is a degree of polynomial f , a_0 is its leading coefficient,

$$x^k = (x_1^k, \dots, x_n^k), \quad W(x^k) = (W_1(x_1^k), \dots, W_n(x_n^k)), \quad W_i(x^k) = \frac{f(x_i^k)}{a_0 \prod_{j \neq i} (x_i^k - x_j^k)}, \quad i = 1, 2, \dots, n.$$

In the middle of the last century, Prof. K. Dochev showed convergence of the iterative method (a) to the vector root (zero) $\xi = (\xi_1, \dots, \xi_n)$ of the polynomial at a specific localization of the initial point (approximation) x^0 :

$$\max \left\{ |x_i^0 - \xi_i|, i = 1, 2, \dots, n \right\} < \frac{\sqrt[n]{2} - 1}{2\sqrt[n]{2} - 1} \min \left\{ |\xi_j - \xi_i|, j \neq i \right\}.$$

In 1964, the Bulgarian mathematicians K. Dochev and P. Burnev presented a simultaneous method for finding simple zeros of the polynomials with third order of convergence (with a unit higher convergence than the method of K. Weierstrass (a)):

$$x^{k+1} = x^k - D(x^k), \quad k = 0, 1, \dots, \quad (b)$$

where:

$$D(x^k) = (D_1(x_1^k), \dots, D_n(x_n^k)), \quad D_i(x^k) = W_i(x^k) \left(1 - \sum_{j \neq i} \frac{W_j(x^k)}{x_i^k - x_j^k} \right), \quad i = 1, 2, \dots, n.$$

A few years later, L. Ehrlich introduced and studied another simultaneous method with the same order of convergence (again for polynomials with prime zeros):

$$x^{k+1} = x^k - E(x^k), \quad k = 0, 1, \dots, \quad (c)$$

$$E(x^k) = (E_1(x_1^k), \dots, E_n(x_n^k)), \quad E_i(x^k) = W_i(x^k) \left(1 + \sum_{j \neq i} \frac{W_j(x^k)}{x_i^k - x_j^k} \right), \quad i = 1, 2, \dots, n.$$

We will note that, finding the form of operator by the operator W, presented above, is due to W. Börsch-Supan (1970). A parametric family of iterative methods for simultaneous approximation of zeros of polynomials was found in [5] of the candidate. The methods of (b) K. Dochev and P. Burnev and (c) L. Ehrlich are included in the family as special cases. The family of iterative methods has the type:

$$x^{k+1} = x^k - I(x^k), \quad k = 0, 1, \dots, \quad (d)$$

$$I(x^k) = (I_1(x_1^k), \dots, I_n(x_n^k)), \quad I_i(x^k) = W_i(x^k) \frac{1 + (\alpha - 1) \sum_{j \neq i} \frac{W_j(x^k)}{x_i^k - x_j^k}}{1 + \alpha \sum_{j \neq i} \frac{W_j(x^k)}{x_i^k - x_j^k}}, \quad i = 1, 2, \dots, n.$$

At the value $\alpha = 0$ of the parameter from the above iterative method, the method of the Bulgarian mathematicians is obtained, and at $\alpha = 1$ - the method of L. Ehrlich. With P. Proinov's theory of convergence of iterative procedures in metric and -dimensional spaces and in particular of his studies on convergence of procedures (a) and (b) Dr. S. Ivanov has received an estimate of the error proposed - method (d).

In 1991, three Japanese mathematicians T. Sakurai, T. Torii, and H. Sugiura introduced an iterative method for simultaneously finding the zeros of a fourth-order polynomial. The iterative procedure has the form

$$x^{k+1} = x^k - \Phi(x^k), \quad k = 0, 1, \dots, \quad (e)$$

where

$$\Phi(x^k) = (\Phi_1(x_1^k), \dots, \Phi_n(x_n^k)), \quad \Phi_i(x^k) = \begin{cases} \frac{2L_i(x^k)}{L_i(x^k) - F_i(x^k)}, & f(x_i^k) \neq 0; \\ 0, & f(x_i^k) = 0 \end{cases},$$

$$L_i(x^k) = \frac{f'(x_i^k)}{f(x_i^k)} - \sum_{j \neq i} \frac{1}{x_i^k - x_j^k}, \quad F_i(x^k) = \frac{f''(x_i^k)}{f(x_i^k)} - \left(\frac{f'(x_i^k)}{f(x_i^k)} \right)^2 + \sum_{j \neq i} \frac{1}{x_i^k - x_j^k}, \quad i = 1, 2, \dots, n.$$

In papers [7], [8], and [9], through the general theory of Prof. P. Proinov on the convergence of iterative procedures of the E. Picard type, two new semilocal convergence theorems for the Sakurai - Torii - Sugiura method were proved. An important preliminary achievement is the localization of the initial point, ensuring convergence of the iterative procedure (e). In one theorem, the initial point must satisfy an inequality of the form:

$$\max \left\{ |x_i^0 - \xi_i|, \quad i = 1, 2, \dots, n \right\} < \frac{2 \min \left\{ |\xi_j - \xi_i|, \quad j \neq i \right\}}{4 + \sqrt{8n - 10}}.$$

In the other theorem, the localization guaranteeing convergence is given by the inequality

$$\max \left\{ |x_i^0 - \xi_i|, \quad i = 1, 2, \dots, n \right\} < \frac{2 \min \left\{ |\xi_j - \xi_i|, \quad j \neq i \right\}}{3 + \sqrt{8n - 7}}.$$

The estimates of convergence (e) are obtained. These results improve and complement the research of M. Petkovic et al.

In [1], the classical iterative method of E. Halley is modified as a method for finding all zeros of a polynomial:

$$x^{k+1} = x^k - H(x^k), \quad k = 0, 1, \dots, \quad (f)$$

where:

$$H(x^k) = (H_1(x_1^k), \dots, H_n(x_n^k)), \quad H_i(x^k) = \frac{f(x_i^k)}{f'(x_i^k)} \left(1 - \frac{1}{2} \frac{f(x_i^k) f''(x_i^k)}{(f'(x_i^k))^2} \right)^{-1}, \quad i = 1, 2, \dots, n.$$

The creation and investigation of the iterative procedure (f) are influenced by a similar approach taken by P. Batra in 2002. In P. Batra's work, the classical Newton method is transformed into a simultaneous method for finding all zeros of a polynomial. Sufficient conditions for convergence of the procedure (f) are obtained in [1], i.e. for convergence of the iterative sequence $\{x^k = (x_1^k, \dots, x_n^k)\}$ to the vector of zeros of the polynomial $\xi = (\xi_1, \dots, \xi_n)$. In the first cubic convergence theorem, it is sufficient to require an initial point $x^0 = (x_1^0, \dots, x_n^0)$ to satisfy the restriction

$$\max \left\{ |x_i^0 - \xi_i|, \quad i = 1, 2, \dots, n \right\} < \frac{2 \min \left\{ |\xi_j - \xi_i|, \quad j \neq i \right\}}{n + 1 + \sqrt{(n-1)(5n-1)}}.$$

The second convergence theorem requires the initial point to have different coordinates and moreover:

$$E(x^0) = \left\| \frac{x^0 - \xi}{\min \{|x_j^0 - x_i^0|, j \neq i\}} \right\|_p < \frac{1}{n}, \quad h(E(x^0)) \geq 0,$$

where $\|\cdot\|_p$ is the traditional p -norm, $p \geq 1$, and the function

$$h(t) = (1-t)(1-nt)(1-bt) - n(n-1)t^2, \quad 1 < b < 2.$$

The p -norm of the difference between the elements of the convergent series and the vector of polynomial zeros in the generalized method (f) is estimated. If $h(E(x^0)) > 0$, then the convergence is cubic

In paper [4], the investigation from [1] of the applicant continues. Modifications of Newton's, Halley's and Chebyshev's methods are considered here, in which the zeros of the polynomials (whether simple or multiple) are found simultaneously. The multiples of the zeros are assumed to be known in advance. A priori and a posteriori estimates of the convergence of these iterative methods have been found. Numerical examples are presented, confirming the theory.

In [3] and [6], a generalized method of E. Schröder for finding all zeros of a polynomial simultaneously was introduced (following the ideas of P. Batra). The method has a quadratic convergence and there is no restriction on the multiplicity of zeros, i.e. the multiplicity of each zero of the polynomial is not required to be known in advance. The iterative procedure has the form

$$x^{k+1} = x^k - S(x^k), \quad k = 0, 1, \dots, \quad (j)$$

$$S(x^k) = (S_1(x_1^k), \dots, S_n(x_n^k)), \quad S_i(x^k) = \left(\frac{f'(x_i^k)}{f(x_i^k)} - \frac{f''(x_i^k)}{f'(x_i^k)} \right)^{-1}, \quad i = 1, 2, \dots, n.$$

The convergence of the iterative scheme is of the second order and is guaranteed with sufficient proximity of the initial point $x^0 = (x_1^0, \dots, x_n^0)$ to the vector $\xi = (\xi_1, \dots, \xi_n)$ of the zeros of the polynomial f . More precisely, the next restrictions are valid:

$$\max \{|x_i^0 - \xi_i|, i = 1, 2, \dots, n\} < \frac{2 \min \{|\xi_j - \xi_i|, j \neq i\}}{n+1 + \sqrt{n^2 + 6n - 7}}.$$

An error estimate is given. In [6], the local convergence theorem is transformed into a semilocal convergence theorem.

In conclusion, I will mention a few facts that make a good impression:

- Good awareness of the author about the current state of scientific knowledge ;
- Complete and accurate evidence of the allegations (I noticed inaccuracies in the notations only in one place). The reader has no doubt in the results achieved;
- Excellent mastery of the used mathematical methods, undoubtedly this is due to the deep knowledge of the auxiliary apparatus;
- Well-arranged research following generally accepted standards.

5. EVALUATION OF THE PERSONAL CONTRIBUTION OF THE CANDIDATE

Most papers with which the candidate participates in the competition are co-authored with other specialists. Only two are independent. I am not informed about the existence of a declaration for internal distribution of the authors' participation in the preparation and publication of the research attached to the competition under discussion. Therefore, I believe that the participation of Dr. S. Ivanov is equivalent to the other co-authors. I do not think that the "arrangement" of the authors in the collective scientific works has anything to do with the degree of their participation in the production of the publication. Therefore, in my opinion, I did not pay attention to this circumstance in the review.

6. CRITICAL NOTES AND RECOMMENDATIONS

I have no critical remarks. All the documents for the competition are prepared accurately and conveniently for review.

It seems to me that the scientific results obtained by the candidates should be systematized in a monograph devoted to interactive methods for finding individual or simultaneously all zeros of polynomials. The monograph will be with serious interest among the scientific community.

7. PERSONAL IMPRESSIONS

I have known my colleague S. Ivanov recently - in connection with the procedure of the commented competition. I will emphasize once again that he is a built university teacher of mathematics (he has practiced the profession for more than 8 years). Undoubtedly, he has a deep knowledge of the theory in which his research is based. I also suspect that the scientific work gives pleasure to the candidate for the academic position of "associate professor", which is a guarantee for future success.

CONCLUSION

The scientific and teaching qualification of the candidate fully corresponds to his claims for the academic position of "associate professor". Dr. S. Ivanov has presented a sufficient number of scientific papers published after the materials used in the defense of the educational and scientific degree "Doctor" and holding the academic position of "Chief Assistant". The textbook is useful for teaching at the University of Plovdiv. The scientific papers have original scientific contributions, most of which have been published in impact factor journals and are reflected in the Web of Science and Scopus databases. The candidate's results have received international recognition, having been cited many times by other authors.

The results achieved by the candidate in the teaching and research activities significantly exceeded the specific minimum requirements of FMI of the University of Plovdiv, adopted in connection with Act on development of the academic staff in the Republic of Bulgaria, the Rules for application of Act on development of the academic staff in the Republic of Bulgaria, and the Rules for the development of the academic staff of Plovdiv University "Paisii Hilendarski".

After getting acquainted with the materials and scientific papers presented for the competition, analysis of their significance, I find reasonable to give my positive assessment and recommend to the Scientific Jury to prepare a report-proposal to the Faculty of Physics and Technology of Plovdiv University "Paisii Hilendarski" Dr. Stoil Ivanov Ivanov at the academic position of "Associate Professor" in the Department of Educational Technologies of the same faculty in the Professional field: 4.5. Mathematics and Scientific specialty: Mathematical analysis.

05.09.2021

Reviewer:
(Prof. DSc Angel Dishliev)