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OPINION

for a competition for an academic position

"Associate professor"

professional field 4.5 Mathematics (Probability Theory and Mathematical Statistics),

for the needs of Plovdiv University "Paisii Hilendarski",

Faculty of Mathematics,

published in SG no. 99 of 20.11.2020 and on the websites of Plovdiv University

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The review was prepared by: Assoc. Prof. Dr. Doncho Stefanov Donchev FMI, Sofia University "St. Kliment Ohridski", in my capacity as a member of the scientific jury of the competition according to Order № P33-636 / 19.02.2021 of the Rector of Plovdiv University.

In the competition for 'associate professor', as a candidate applies Ch. Assistant Professor Dr. Petar Ivanov Kopanov from Plovdiv University "Paisii Hilendarski".

1. General description of the submitted materials:

Submitted is a set of documents which consist of:

- Articles - 21 issues

- References for citations - 5 screenshots of web sites

The submitted set is in accordance with the Regulations for the development of the academic staff of the Plovdiv University. Four of the articles are published in Bulgarian journals, 17 in international journals.

2. General description of the applicant's activity:

For educational and pedagogical activity and preparation of the candidate I have direct impressions from the time when I worked on a civil contract at the University of Plovdiv in 2001 and 2002. My opinion is that he is an experienced instructor who knows how to motivate students.

3. Evaluation of the scientific and scientific-applied activity of the candidate

For participation in the competition are presented 21 papers, of which as follows :. articles - 19; reports - 2; popular publications - 1. Of the presented articles with impact factor are 12. Four of the articles have been published in national journals, and the rest in referenced international journals. One of the publications is in Russian and the others in English; 6 of the

works are with one co-author, 2 with two co-authors and 8 with three co-authors. The candidate is a single author of 5 articles.

Content analysis of the works for which I can give an expert opinion.

Papers 11, 14, 15 and 16 are devoted stability issues of impulse SDUs, in which the impulses act on a time interval of a positive length, and the times between two sequential impulses are random. The resulting stochastic processes belong to the well-known class of piece-wise deterministic processes. These processes were a subject of intensive research in the 1980s, with a significant contribution to the development of the general theory of a renowned scientists like M. Davis and A. Yushkevich. In the early 1990s, the first of them published a book with the same title: "Piece-Wise Deterministic Processes". For them random are only the moments of their jumps, and in the position of the process immediately after the jumps. Between jumps, their sample paths are deterministic (i.e. they can satisfy the SDU) and start from the position of the process immediately after the jumps. The processes studied in these articles are a special case of the described scheme, since only the intervals between the jumps are random, whereas the position of the process immediately after the jumps is deterministic.

It is assumed that the intervals between the pulses have a Gamma distribution, or special cases of this distribution (i.e. Erlang distribution). The articles are joint research with specialists in differential equations, and I assume that the candidate's contribution concerns the probabilistic part of the articles in which some properties of the Gamma distribution are obtained.

Articles 8 and 20 are devoted to the problem of uniqueness of the distribution of a random variable assuming that moments of any order are given. The articles are joint with prominent specialists in this field. In Paper 8, applications of the Linn condition are considered, which guarantees the uniqueness of the distribution in terms of the asymptotic behavior of the logarithmic derivative of the probability density. Conditions are obtained under which this condition is inherited by products and degrees of random variables for which it holds. In paper 20, new sufficient conditions for uniqueness are obtained which do not depend on density's derivative . These conditions are in the form of integral criteria and refer to symmetric distributions defined on the whole real line, as well as to distributions whose support the positive part of the real line.

In preparing my opinion, I paid special attention to work 6. It originates to a classical problem studied by De Rham in the 1950s. De Rham's main results are summarized in the seminal paper:

Georges De Rham, Sur quelques courbes définies par des équations fonctionelles, Rendiconti del Seminario Matematico dell'Universita e del Politecnio di Torino, 101–113, 1957,

however his first publications concerning some particular cases appeared in 1953. In the cited article, De Rham derives both equations that appear in Propositions 1.1 and 1.2, and today they are known as De Rham's functional equations. The singular nature of the distribution functions $F_p(x)$ for $p \neq 1/2$, is also established in this paper, i.e. it is shown that the function $F_p(x)$ has derivative 0 almost everywhere, and that it does not exist only in dyadic points (the last fact in article 6 is the claim of Theorem 5). In view of all this, as a main result of article 6, I consider Theorem 6, which gives a criterion when the derivative of the function $F_p(x)$ is equal to 0, ∞, or does not exist. In my opinion, however, this criterion contradicts the strong law of the large numbers of J. Bernoulli. According to this law, the measure with distribution function $F_p(x)$ is supported by the set $\{x : \liminf k_n(x) = \limsup k_n(x) = p\}$.

Since the Lebesque measure of this set is 0 (due to the singularity of $F_p(x)$), the density $f_p(x)$ should be equal to ∞ on this set.

However, Theorem 6 implies that $f_p(x)$ is infinite for all x such that

$$\liminf k_n(x) = \limsup k_n(x) > -\frac{\log(2(1-p))}{\log p - \log(1-p)}, p > 1/2$$

$$\liminf k_n(x) = \limsup k_n(x) < -\frac{\log(2(1-p))}{\log p - \log(1-p)}, p < 1/2$$

4. **Assessment of the candidate's personal contribution**

I assume that the candidate's contribution to the joint papers is equal.

5. **Critical remarks and recommendations**

My critical remarks confine to my comment on article 6.

6**. Personal impressions**

My personal impressions of P. Kopanov as a colleague, scientist and instructor are very good.

CONCLUSION

The documents and materials submitted by Petar Ivanov Kopanov meet the requirements of the Law for the Development of the Academic Staff in the Republic of Bulgaria (ZRASRB),

the Regulations for implementation of ZRASRB and the Regulations of PU "Paisii Hilendarski".

The candidate has submitted a sufficient number of scientific papers published after the materials used in the defense of ONS 'Doctor'. In the works of the candidate there are original scientific and applied contributions, which have received international recognition as a representative part of them are published in journals and scientific journals published by international academic publishers.

The results achieved by Petar Ivanov Kopanov in the teaching and research activities fully comply with the specific requirements of the Faculty of Mathematics, adopted in connection with the Regulations of the University of Plovdiv for the application of ZRASRB.

After getting acquainted with the materials and scientific papers presented in the competition, analysis of their significance and contained in them scientific, scientific-applied and applied contributions, I find it reasonable to give my positive assessment and recommend to the Scientific Jury to prepare a report-proposal to the Faculty. Faculty Council. in Mathematics for the election of Petar Ivanov Kopanov to the academic position of 'Associate Professor' at the University of Plovdiv "P. Hilendarski" in the professional field of Probability Theory and Mathematical Statistics.

03.04.2021

Ass. Prof. Dr. Doncho Donchev