
**ANNOTATIONS OF THE PUBLICATIONS INCLUDED IN THE DOCUMENTS
FOR THE PROCEDURE**

- (1) B. Zlatanov: ON WEAK UNIFORM NORMAL STRUCTURE IN WEIGHTED ORLICZ SEQUENCE SPACES, *J. Math. Anal. Appl.* **341** (2008) 1042–1054

The weakly convergent sequence coefficient $WCS(X)$ of a Banach space was introduced by Bynum. It is closely related with the normal structure of X , the Banach–Mazur distance, and the fixed point property. Thus $WCS(X)$ is interesting to be known. The value of WCS is known for ℓ_p , c_0 and for Hilbert spaces. Cui presented a formula for calculating it in the Köthe sequence space. He calculated WCS for Orlicz sequence spaces equipped either with the Luxemburg’s or Amemiya’s norm, provided that M satisfies the Δ_2 condition. We have investigated the WCS for weighted Orlicz sequences spaces, equipped either with the Luxemburg’s or Amemiya’s norm, and a weight sequence $w = \{w_n\}_{n=1}^\infty$ belonging to the classes either Λ or Λ_∞ . The weight sequence is from the class Λ_∞ if it is nondecreasing and convergent to infinity, and it is from the class Λ if it has a convergent to zero subsequence $\{w_{n_k}\}_{k=1}^\infty$, such that $\sum_{k=1}^\infty w_{n_k} = \infty$. We have proved that a weighted Orlicz sequence space $\ell_M(w)$, equipped with either the Luxemburg or Amemiya norm has weak uniform normal structure if and only if $\ell_M(w) \cong h_M(w)$ for a wide class of weight sequences $w = \{w_n\}_{n=1}^\infty$ (if $w = \{w_n\}_{n=1}^\infty$ belonging to the classes either Λ or Λ_∞). In the auxiliary results we characterize the weakly null sequences for the investigated classes of Orlicz spaces. An example is constructed, where the Orlicz function M does not have the Δ_2 -condition but by choosing a suitable weight sequence $\lim_{n \rightarrow \infty} w_n = \infty$ we get that $\ell_M(w)$ has a weak uniform normal structure.

- (2) B. Zlatanov: ON EQUIVALENT ANALYTIC NORMS IN ORLICZ-LORENTZ SEQUENCE SPACES, *Plodiv University, "Paissii Hilendarski", Bulgaria Scientific Works*, **36**, (2009) 115-128.

Leung proved that an Orlicz sequence space is isomorphically polyhedral if $\lim_{t \rightarrow 0} \frac{M(\lambda t)}{M(t)} = \infty$. Dew has shown that if a Muscelak–Orlicz h_Φ sequence space is stabilized asymptotic ℓ_∞ with respect to the unit vector basis, then it is isomorphically polyhedral. Using the ideas of Leung and Dew we prove that if the generating Orlicz function M does not have the Δ_2 -condition at zero, then the existence of an equivalent analytic norm in the Orlicz–Lorentz sequence space $d_0(w, M)$ is equivalent to $d_0(w, M)$ to be isomorphically polyhedral. We show that if $\lim_{n \rightarrow 0} \frac{M(\lambda t)}{M(t)} = \infty$ for some $\lambda > 1$ then the Orlicz–Lorentz sequence space $d_0(w, M)$ is isomorphic to a polyhedral Banach space and therefore it admits an equivalent analytic norm, it is c_0 -saturated and it has a separable dual. We characterize all the c_0 -saturated Orlicz–Lorentz sequence spaces in terms either of the generating Orlicz function or of the existence of an isomorphic copies of ℓ_p . We have presented a class of Orlicz–Lorentz sequence spaces, which is isomorphically polyhedral, admits an equivalent analytic norm and it is c_0 -saturated.

- (3) B. Zlatanov: ON MUSIELAK-ORLICZ SEQUENCE SPACES WITH AN ASYMPTOTIC ℓ_∞ DUAL, *Annaire de l'Universite de Sofia "St. Kliment Ohridski"*, **99**, (2009), 203–214

The notion of asymptotic ℓ_p spaces was defined by Milman and Tomczak–Jaegermann, where the collection of spaces that are now known as stabilized asymptotic ℓ_p spaces. Later Maurey, Milman and Tomczak–Jaegermann presented a more general collection of spaces, known as asymptotic ℓ_p spaces. A complete characterization of the bounded relatively weakly compact subsets $K \subset \ell_M$ is given by Alexopoulos, without using the technique of stabilized asymptotic ℓ_∞ , but only by the properties of the complementary function to the generating Orlicz function M . We investigate Musielak–Orlicz sequence spaces ℓ_Φ with a dual ℓ_Φ^* , which is stabilized asymptotic ℓ_∞ with respect to the unit vector basis. We give a complete characterization of the bounded relatively weakly compact subsets $K \subset \ell_\Phi$. We prove that ℓ_Φ is saturated with asymptotically isometric copies of ℓ_1 and thus ℓ_Φ fails the fixed point property for closed, bounded convex sets and non-expansive (or contractive) maps on them. We illustrate the results with examples.

- (4) B. Zlatanov: ON ANOTHER PROOF OF THE SCHUR PROPERTY IN MUSIELAK–ORLICZ SEQUENCE SPACES, *Plodiv University Paissii Hilendarski, Bulgaria Scientific Works* **37**, (2010) 135–142.

The technique of stabilized asymptotic ℓ_∞ spaces turns to be very useful in the investigation of some properties of Musielak–Orlicz sequence spaces as it is shown by Zlatanov in a previous article. It is well known that any weakly null sequence in ℓ_1 is norm null. This property of ℓ_1 is referred to as the Schur property. Zlatanov has proven in [B. Zlatanov. Schur property and lp isomorphic copies in Musielak–Orlicz sequence spaces, *Bulletin of the Australian Mathematical Society* 75, (2007) 193–210] that if the Musielak–Orlicz sequences space ℓ_Φ is generated by a Musielak–Orlicz function, which satisfies the δ_2 condition and its dual space ℓ_Ψ is a stabilized asymptotic ℓ_∞ space with respect to the unit vector basis, then ℓ_Φ has the Schur property. We present a proof of the above mentioned result with the classical technique of functional analysis, which goes back to the times of Banach.

- (5) B. Zlatanov: ON A CLASS OF KÖTHE SEQUENCE SPACES WITH NORMAL STRUCTURE, *Acta Mathematica Scientia*, 31B(2), (2011), 576–590

We obtain necessary conditions for a Köthe sequence space, with a boundedly complete and shrinking basis to have a normal structure. Following the ideas of Changsen and Fenghui, who have defined a generalized modulus of convexity $\delta_X^{(\lambda)}$ we define a generalized modulus of smoothness $\rho_X^{(\lambda)}$. We show that these generalized moduli are connected in a similar fashion as the classical ones, which is a generalization of Lindenstrauss’ result. We investigate some properties of these new moduli. We obtain estimates of these moduli for any arbitrary Banach space and for $X = \ell_p$. We obtain inequalities between the *WCS* coefficient of a Köthe sequence space X and $\delta_X^{(\lambda)}$. We give sufficient conditions, connected with the modulus of convexity, that ensure that the space has a normal structure. These results are more general than the result of Gao and Lau. An easy to apply result in a wide class of Köthe sequence spaces X is proven: if for some $\varepsilon \in (0, \frac{9}{10})$ holds $\delta_X(\varepsilon) > \frac{1}{3} \left(1 - \frac{\sqrt{3}}{2}\right)$, then X has a normal structure.

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- (6) B. Zlatanov: SOME EXPRESSIONS FOR THE RIESZ ANGLE OF WEIGHTED ORLICZ SEQUENCE SPACES, *Mathematical Sciences* (2013) 7:13

Borwein and Sims defined the Riesz angle $\alpha(X)$ in a Banach lattice. They present applications of the Riesz angle in the investigations of the geometry of Banach spaces. The Riesz angle of ℓ_p and of the Orlicz sequence spaces ℓ_M are known. We obtain an expression for computation of the Riesz angle in weighted Orlicz sequence spaces, generated by an Orlicz function M , satisfying the Δ_2 -condition and a weighted sequence $w = \{w_n\}_{n=1}^\infty$ belonging to the class Λ . Following the ideas of Yan, we give a formula for calculating of the Riesz angle in weighted Orlicz sequence spaces $\ell_M(w)$ and equipped with the Luxemburg's norm. In the case of Amemiya's norm we get only estimates from below and above for the value of the Riesz angle. We present formulas for calculations of the Riesz angle equipped with the Luxemburg's norm, depending only on the behavior of the Orlicz function M . We present several examples of Orlicz functions, that are classical, but not trivial, when it is possible to calculate the exact value of the Riesz angle.

- (7) B. Zlatanov: KOTTMAN'S CONSTANT, PACKING CONSTANT AND RIESZ ANGLE IN SOME CLASSES OF KÖTHE SEQUENCE SPACES, *Carpathian J. Math.*, **35**(1) (2019), 103–124

The packing constant and the Kottman constant, defined by Kottman, are interesting and very important parameters for studying of the geometry structure of Banach spaces. The packing constant is known for the classical sequence spaces ℓ_p , Orlicz, Nakano, Musciela–Orlicz, Lorentz, Orlicz–Lorenz and Cesaro. Kottman gets a relation between the packing constant and the Kottman's constant. We present different formula for calculating of the packing constant and thus the packing constant in weighted Orlicz sequence spaces with a weight sequence $w = \{w_n\}_{n=1}^\infty$ belonging to the class Λ , equipped with the Luxemburg's or p -Amemiya's norm. This formula is different from the known one for calculating of the Kottman's constant and a bit easier. Thus knowing the packing constant we can calculate the Kottman's constant in Musciela–Orlicz sequence spaces. Unfortunately, the Riesz angle is much more difficult to calculate. We prove that for a wide class of Köthe sequence spaces the Kottman's constant and the Riesz angle are equal. The classes of these spaces include either those that are order continuous with the Fatou property or those that the unit vector basis is unconditional and boundedly complete. We get some geometric properties connected with the Riesz angle. Knowing the packing constant, we thus find the Kottman's constant for ℓ_p , Orlicz, weighted Orlicz, Nakano, Musciela–Orlicz, Lorentz and Cesaro sequence spaces. We give the exact value of the Riesz angle also in the cases when M does not have the Δ_2 -condition. We present some relations between the Riesz angle and the moduli of convexity and smoothness in Köthe sequence spaces and some relations between the Riesz angle and coefficients connected only to the generating Orlicz function in Orlicz spaces and Lorentz–Orlicz spaces.

- (8) M. Petric, B. Zlatanov: FIXED POINT THEOREMS OF KANNAN TYPE FOR CYCLICAL CONTRACTIVE CONDITIONS, *Plodiv University, Faculty of Mathematics and Informatics, REMIA, December* (2010), 187–194

The main aim of this paper is to obtain fixed point theorems for Kannan and Zamfirescu operators in the presence of a cyclical contractive condition. A method for the approximation of the fixed points is also provided, for which both a priori and a posteriori error estimates are given. Our results generalize, unify and extend several important fixed points theorems in the literature. In order to illustrate the efficiency of our generalizations, five significant examples are also given.

- (9) M. Petric, B. Zlatanov: BEST PROXIMITY POINTS AND FIXED POINTS FOR p -SUMMING MAPS, *Fixed Point Theory and Applications* 2012, 2012:86 (2012) doi:10.1186/1687-1812-2012-86.

The idea to consider a cyclic map between 2 closed convex not intersecting sets started with the result of Eldred and Veermani. They defined a new type of points, the best proximity point of a map in a set, which generalizes the notion of fixed points. Later Karpagam and Agrawal generalized this idea for cyclic maps between p sets. The condition imposed on the map seems to be too restrictive, as far as it is satisfied only when the distances between the consecutive sets are equal. We have further generalized the idea of cyclic maps by defining the notion of p -summing maps. This new type of a contractive condition ensures the existence and uniqueness of fixed points and best proximity points in uniformly convex Banach spaces and the distances between the consecutive sets may be different. The results of the four mentioned authors are corollaries of the main result. We have also obtained a result about fixed points for cyclic summing maps. We provide examples to validate our results and we have shown that there exists p -cyclic maps, such that the distances between the sets are different.

- (10) S. Karaibyamov, B. Zlatanov: FIXED POINTS FOR MAPPINGS WITH A CONTRACTIVE ITERATE AT EACH POINT, *Mathematica Slovaca* **64**(2), (2014) 455-468

We generalize the results of Sehgal and Guseman for mappings on a complete metric space with a contractive iterate condition at each point. The results of Sehgal and of Guseman are a particular case of our main result. We have illustrated the main theorem with different examples. One of them is about a map in an arbitrary Banach space.

- (11) B. Zlatanov: BEST PROXIMITY POINTS FOR p -SUMMING CYCLIC ORBITAL MEIR-KEELER CONTRACTIONS, *Nonlinear Analysis: Modelling and Control* **20**(4), (2015) 528-544

The idea to generalize orbital contractions for cyclic Meir-Keeler maps was introduced by Karpagam and Agrawal. We have introduced the notion of p -summing orbital Meir-Keeler contractions. Contrary to the idea of considering a p -summing cyclic contraction, we were not able to get a single condition that will ensure the uniqueness and existence of best proximity points. By proposing two p -summing conditions we get necessary and sufficient conditions for the existence and uniqueness of best proximity points. The results for best proximity points for cyclic Meir-Keeler orbital contractions, obtained by Karpagam and Agrawal, are a particular case of the main theorem. We illustrate the main result with an example.

- (12) B. Zlatanov: BEST PROXIMITY POINTS IN MODULAR FUNCTION SPACES, *Arabian Journal of Mathematics*, **4**(3), (2015) 215-227.

An idea for the generalization of fixed points is by changing the underlying space. Khamsi, Kozłowski and Reich were the first to present such a generalization in modular function spaces. Further, mainly Kazlovski and Kamshi, together with their collaborates, continue the investigation of the geometry of these spaces and on a fixed point theory in them. We generalize the notion of best proximity points in the context of modular function spaces. We find sufficient conditions for the existence and uniqueness of best proximity points for cyclic maps in modular function spaces, by replacing the uniform convexity with the property $UC1$. It is interesting that in modular function spaces there are different notions that generalize the uniform convexity of a Banach space. We needed to generalize the key lemmas of Eldred and Veermani for convergence of sequences in the case of modular function spaces. These results enrich the knowledge of the geometry of the modular function spaces, which may have very strange structure. We present an easy to apply corollary, where the Δ_2 property was replaced by a much easier one to check a condition. We present an application of the main result for cyclic integral operators in Orlicz function spaces, endowed with an Orlicz function modular and we get sufficient conditions, which ensure the existence of a unique solution, that depend only on the generating Orlicz function.

- (13) M. Ivanov, B.Zlatanov, N. Zlateva: A VARIATIONAL PRINCIPLE AND BEST PROXIMITY POINTS, *Acta Mathematica Sinica, English Series*, **31**(8), (2015) 1315-1326.

Ekeland introduced a variational principle and he presented applications in different fields of mathematics. Since his publications there have been a great number of generalizations and applications. A widely used application of the variational technique is the the theory of fixed points. There were no results about variational technique in best proximity points. We generalize Ekeland's Variational Principle for cyclic maps. We present applications of this version of the variational principle for proving of the existence and uniqueness of best proximity points for different classes of cyclic maps, namely cyclic contractions, Reich maps, Kannan, Ćirić, Hardy and Rogers, Chatterjee, Zamfirescu and iterated contractions.

- (14) B. Zlatanov: ERROR ESTIMATES FOR APPROXIMATING BEST PROXIMITY POINTS FOR CYCLIC CONTRACTIVE MAPS, *Carpathian Journal of mathematics* **32**(2), (2016) 265-270

In contrast with all the results about fixed points for self maps and cyclic maps, where "a priori error estimates" and "a posteriori error estimates" are obtained, there are no such results about best proximity points. We have filled this gap for cyclic contractions, provided that the underlying space is uniformly convex with modulus of convexity of power type. We have illustrated the result by an example

- (15) Saravanan Karpagam, B. Zlatanov: BEST PROXIMITY POINTS OF p -CYCLIC ORBITAL MEIR-KEELER CONTRACTION MAPS, *Nonlinear Analysis: Modelling and Control* **21**(6), (2016) 790-806

Using the technique of L -functions, introduced by Lim and further developed by Suzuki, we get sufficient conditions for the existence and uniqueness of best proximity points and fixed

points for p -cyclic, orbital Meir–Keeler maps. We illustrate the obtained results with different examples.

- (16) R. Koleva, B. Zlatanov: ON FIXED POINTS FOR CHATTERJEA’S MAPS IN b -METRIC SPACES, *Turkish Journal of Analysis and Number Theory* **4**(2), (2016) 31–34

A kind of generalizing fixed point results can be obtained by changing the underlying space. Bakhtin considered the underlying space to be a b -metric space. By using the technique of the consecutive iterations we have to impose some restrictions on the b -metric constant. We have realized that b -metric spaces have a lot in common with the modular functions spaces. Thus we find sufficient conditions for the existence and uniqueness of fixed points of Chatterjea’s maps in b -metric space. These conditions do not involve the b -metric constant, but we require that the set of the orbits of the successive iterations is bounded, a condition widely used in the fixed point theory in modular function spaces. We establish a priori error estimate for the sequence of successive iterations. The error estimate, which we present, is better than the well known one for a wide class of Chatterjea’s maps in metric spaces. We give some examples that show that our results are more general than the known ones.

- (17) A. Ilchev, B. Zlatanov: ON FIXED POINTS FOR REICH MAPS IN b -METRIC SPACES, *Annual of Konstantin Preslavski University of Shumen, Faculty of Mathematics and Computer science* **XVII C**, (2016) 77–88

We have continued the investigation on fixed point results in b -metric spaces by generalizing the results of Koleva and Zlatanov. In this paper we find sufficient conditions for the existence and uniqueness of fixed points for a class of Reich maps in b -metric space. These conditions do not involve the b -metric constant, but we require that the set of the orbits of the successive iterations is bounded, a condition widely used in the fixed point theory in modular function spaces. We establish a priori error estimate for the sequence of successive iterations. The results of Koleva and Zlatanov turn to be corollaries of the main results. The error estimate, which we present, is better than the well-known one for a wide class of Reich maps in metric spaces. We have illustrated the main theorem with an example.

- (18) A. Ilchev, B. Zlatanov: FIXED AND BEST PROXIMITY POINTS FOR KANNAN CYCLIC CONTRACTIONS IN MODULAR FUNCTION SPACES, *Journal of Fixed Point Theory Applications* **19**(4), (2017) 2873–22893

The first generalization of best proximity points in modular function spaces was made by Zlatanov. We generalize the notion of best proximity points and the notion of fixed points for cyclic contraction maps in modular function spaces about Kannan maps. We have found sufficient conditions for the existence and uniqueness of best proximity points and fixed points for cyclic Kannan maps in modular function spaces. As far as modular function spaces can sometime have strange geometry we needed to generalize some of the results of Eldred and Veermani about the convergence of sequences. These results enrich the knowledge on the geometry of the modular functions spaces. As corollaries we get sufficient conditions for the

existence and uniqueness of best proximity points and fixed points for cyclic maps in Orlicz spaces, endowed with an Orlicz function modular. We present an application of the results for solving integral equations by applying the idea of best proximity points. We have illustrated the last mentioned result by particular examples.

- (19) Mihaela Petric, Boyan Zlatanov: BEST PROXIMITY POINTS FOR p -CYCLIC SUMMING ITERATED CONTRACTIONS. *FILOMAT*. **32**(9) (2018) 3275–3287

Following the ideas of Rheinboldt for iterated contractive conditions we generalize the p -summing contractions maps in the context of iterated p -summing contractions. We find sufficient conditions for these new types of maps, that ensure the existence and uniqueness of best proximity points in uniformly convex Banach spaces. We have obtained sufficient conditions for the existence and uniqueness of best proximity points in reflexive Banach spaces for iterated p -summing contractions that generalize the results of Thagafi and Shahzad. As an application we show that the results of Petric and Zlatanov for p -summing maps, Karpagam and Agrawal for p -cyclic contraction maps and Petric for p -cyclic Kannan contractions are corollaries of the main result. We present a generalization for p -cyclic iterated summing Kannan maps. We were able to get sufficient conditions for the existence of best proximity points only in the case when the Kannan contraction constant is smaller than or equal to $1/4$. By imposing a second condition we get the same result, provided that the Kannan contraction constant is smaller than or equal to $1/2$. We give some results about the p -cyclic summing Chatterjea contraction. We get results about 3-cyclic maps, without any additional conditions and for p -cyclic maps with an additional condition similar to that about the Kannan maps. These observations show that when considering Kannan or Chatterjea contraction in the context of summing maps, that may be very different from the classical contractions. We also do all of the above mention generalizations in the case when the underlying space is only a reflexive Banach space.

- (20) Tharmalingam Gunasekar, Saravanan Karpagam, B. Zlatanov: ON p -CYCLIC ORBITAL M-K CONTRACTIONS IN A PARTIAL METRIC SPACE, *Mathematics*, **6**(7), (2018), 116

Continuing the idea to generalize best proximity results by changing the underlying space we have considered maps defined in a partially ordered complete metric space by defining the property UC . The idea to consider the property UC in metric spaces is due to Suzuki, Kikkawa and Vetro. They have replaced the key lemmas of Eldred and Veermai in a uniformly convex Banach space with the property UC . We generalize Meir-Keeler contractions for partially ordered metric spaces. Sufficient conditions for the existence and uniqueness of fixed points and the best proximity points for these maps in complete partial metric spaces are obtained. In order to get some applications and examples we obtain a necessary and sufficient condition for the completeness of partial metric spaces. We get sufficient conditions for a partial metric space to be either Hausdorff or a normal topological space. The results are illustrated with an example.

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- (21) M.L.Suresh, T. Gunasekar, S. Karpagam, B. Zlatanov: A STUDY ON p -CYCLIC ORBITAL GERAGHTY TYPE CONTRACTIONS, *International Journal of Engineering & Technology*, **7**(4.10) (2018) 883 -887

We introduce a p -cyclic orbital Geraghty type of contraction $S : \cup_{i=1}^p B_i \rightarrow \cup_{i=1}^p B_i$, provided that (X, ρ) be a metric space and B_1, B_2, \dots, B_p be nonempty subsets of X . Convergence of a unique fixed point and a best proximity point for this map is obtained in a uniformly convex Banach space setting. Also, this best proximity point is the unique periodic point of such a map.

- (22) A. Ilchev, B. Zlatanov: ERROR ESTIMATES OF BEST PROXIMITY POINTS FOR REICH MAPS IN UNIFORMLY CONVEX BANACH SPACES, *Annual of Konstantin Preslavsky University of Shumen Faculty of Mathematics and Informatics*, **XIX C**, (2018), 3–20.

We obtain sufficient conditions for the existence and uniqueness of best proximity points for cyclic Reich contraction maps and we find a priori and a posteriori error estimates of the best proximity point, obtained by the process of successive iterations associated to a cyclic Reich contraction map, which is defined on a uniformly convex Banach space with modulus of convexity of power type. Our results generalize known results about the cyclic Kannan type and the Chatterjea type of maps. As corollaries we get a priori and a posteriori error estimates of the best proximity point, obtained by the process of successive iterations associated to the cyclic Kannan type and the Chatterjea type of maps.

- (23) S. Bozhkov, K. Kolikov, B. Zlatanov: ESTIMATES OF THE CORRECTION COEFFICIENT IN COULOMB'S LAW FOR ELECTROSTATIC INTERACTION BETWEEN TWO CHARGED CONDUCTING SPHERES, *Bulletin of the Transilvania University of Brasov, Series III - Mathematics, Informatics, Physics*, **8**(57)(1), (2015) 1-14.

In the present work we consider the coefficient (correction coefficient), which compliments Coulomb's Law in the case of an electrostatic interaction between two charged conducting spheres with equal radii and charges. It is proved that the correction coefficient is smaller than one, when the ratio of the radii to the distance between their centers is smaller than $\frac{2}{5}$. We have obtained a formula for calculating the correction coefficient with an arbitrary precision with the help of partial sums.

- (24) Б. Златанов, С. Караибрямов, Б. Царева. Вертикална интеграция на обучението в средното училище и университета чрез проективни методи в динамична среда *Математика + 1*, (2012), 50-60

The authors of the article have embedded new elements for DGS "infinite point" and a new function "swap of finite & infinite points". This function allows the users to swap points and get new constructions. This gives the advantage that any task involving a complete quadrangle can be considered either as a parallelogram, a trapezoid or an arbitrary quadrangle. We examine the application of three remarkable theorems of projective geometry, ensuring vertical integration of the secondary and tertiary education. Each of them provides a short and versatile

way to solve large groups of tasks and provides the opportunity to generate new tasks. The key element of the approach is to discover the common projective root of large groups of tasks. This allows to generalize results from the textbooks. The new statements are easy for university students and understandable for pupils at school. Knowing the tools through which generalizations are made teaches future educators how to create new school assignments. Preliminary studies in this direction as well as the learning process are greatly facilitated by choosing the right dynamic environment. As can be expected and traced in the present work, the new feature "infinite point" and the new function "swap of finite & infinite points" plays a crucial role. We used DGS Sam, designed and adapted to the needs of synthetic geometry training by the authors of the article, because it has this special function. We illustrate Sam's potential with a few tasks (as far as the authors know, the summaries presented are new).

- (25) B. Zlatanov, S. Karaibryamov, B. Tsareva: ON A NEW FUNCTION IN THE DYNAMIC SOFTWARE, *Practical Seminar on the Project Fibonacci*, Borovets 9-12.04.2012, 133-138.

The Dynamic Geometric Software Sam (written in C # in the middle of the NET Framework 4) was created as a educational software for the needs of the topic of Synthetic geometry. The discovery of the common projective root for large groups of problems is at the heart of our innovative approach, which is relieved by DGS Sam. This allows generalization of tasks related to school mathematics. The new tasks are easy for university students and are understandable to pupils at school. The new function " Swap finite & infinite points" in the menu of DGS Sam optimizes the drawings; it also develops a research style of thinking in students, which is an important element in the modernization of teaching mathematics. For example, the parallelepiped edges are linked to three endless points U_∞ , V_∞ , W_∞ . The swapping of U_∞ , V_∞ , W_∞ with the free endpoints U , V , W allows the user to convert the parallelepiped in no time into a prism, a truncated pyramid, a pyramid with a base parallelogram, a trapezium, a quadrangle or a triangle, with a particular position on a selected surrounding edge. The constructions made for one of the bodies are transferred into a new one. We illustrate the potential of DGD Sam with several tasks (as fas as the authors know, the summaries presented are new).

- (26) S. Karaibryamov, B. Tsareva, B. Zlatanov: EDUCATIONAL SOFTWARE FOR INTERACTIVE TRAINING OF STUDENTS ON THE THEME "MUTUAL INTERSECTING OF PYRAMIDS AND PRISMS IN AXONOMETRY", *Acta Didactica Napocensia* 5(1), (2012) 29-44

In the article, we share our experience of using DGS Sam's for interactive learning on the topic of "Mutual intersecting of pyramids and prisms in axonometry". The program consists of three modules (Teacher, Student and Autopilot), which allow students to be trained on the full range of tasks in the shortest time. A new result on the investigated topic is the classification of the piercing points of the surrounding edges of one of the bodies with the other body, which is based on the mutual arrangement of their auxiliary planes. The solutions of the tasks can be traced to the five classic axonometric projections (cabinet, cavalier prospective, military perspective, increased orthogonal dimertry, increased orthogonal isometry) both in their basic

stages and step by step. Each stage of the decision is accompanied by a comment. Students can manipulate each object of the drawing or apply rotation, translation, scaling, etc. The student can observe the rotating 3D image of the composition of the two intersecting bodies from all sides or separate the rotating 3D image into their common parts. The software allows students not only to learn the matter deeply and in detail for a short time but also to prepare creative course projects such as improvisations on one or a group of tasks from the textbooks and to create original tasks on the topic. We have illustrated the use of the Sam software with a number of different examples that cover all the cases described in the classification of the piercing points.

- (27) S. Karaibryamov, B. Tsareva, B. Zlatanov: OPTIMIZATION OF THE COURSES IN GEOMETRY BY THE USAGE OF DYNAMIC GEOMETRY SOFTWARE SAM, *The Electronic Journal of Mathematics and Technology*, **7**(1), (2013) 22–51

We present a new approach to the teaching of Synthetic Geometry in schools and universities with the help of DGS. The introduction of new elements into DGS helps to optimize the teaching process. These new elements are infinite points in the extended Euclidean plane and the "Swap finite & infinite points" function. We give examples of the usage of these new elements in Projectivity, Homology, Conic Sections, Plane Sections of Polyhedra, and in the application of Pappus' Theorem and Desargues' Theorem. These new features increase the benefits of DGS in teaching and learning Geometry. We optimize the education process by saving time involved in drawing, generalizing large groups of problems, stimulating and helping investigations, and forming a creative style of thinking.

- (28) B. Zlatanov: SOME PROPERTIES OF REFLECTION OF QUADRANGLE ABOUT POINT, *Annals. Computer Science Series*, **11**(1), (2013) 79–91

We show the power of the simultaneous usage of GeoGebra and Maple for generalizing and proving of geometry problems. We present a simple school problem, where, with the help of the dynamics in GeoGebra, new geometric properties are recognized and then we prove them with the help of Maple. We state an open problem for investigation. We suggest a new construction for GeoGebra that can optimize the construction process in the extended Euclidean plane.

- (29) B. Zlatanov: AN ETUDE ON ONE SHARYGIN'S PROBLEM, *Global Journal of Advanced Research on Classical and Modern Geometries*, **3**(2), (2014) 50-61

By the methods of the synthetic geometry we investigate properties of objects generated from a complete quadrangle and a line, which lies in its plane. We start with a problem from the book of Sharygin "Problems in Plane Geometry". We generalize this problem with the help of Pappus', Desargues' and Pascal's Theorems and we discover new concurrent lines, collinear points, and conic sections.

- (30) B. Tsareva, B. Zlatanov: INTERSECTION OF POLYHEDRONS AND A PLANE WITH GEOGEBRA, *North American GeoGebra Journal*, **5**(1), (2016) 39–52

Using GeoGebra, we present an innovative method for teaching of the intersection of polyhedrons with a plane using infinite points and the swap of finite and infinite points. The method presented is efficient and powerful, allowing one to generate solutions of a whole set of problems by solving one instance and using a pre-made applet at any stage of the solution process.

- (31) B. Zlatanov: ON A FAMILY CURVES OF THE SECOND CLASS, *Global Journal of Advanced Research on Classical and Modern Geometries*, 6(2), (2017) 91-105

There are a number of tasks from the school course in geometry, in which are classified very interesting new relations between the objects or a possibility to generate new geometric figures. With the help of DGS, Pappus and Desargues Theorems we discover a set of curves of the second class (conics) connected with the conditions of one school task.

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