REVIEW

Of Materials Submitted for Participation in Contest
For Occupying Academic Position Full Professor
At Plovdiv University Paisii Hilendarski
Candidate: Ass. Prof. Boyan Georgiev Zlatanov, PhD
Reviewer: Prof. Nadezhda Kostadinova Ribarska, DSci
September 10th, 2019

Associate Professor Boyan Georgiev Zlatanov, PhD participates as only candidate in the contest for occupying academic position full professor under higher education area 4. Natural Sciences, Mathematics and Informatics, Professional Field 4.5. Mathematics (Mathematical Analysis) at Plovdiv University Paisii Hilendarski, announced in State Gazette issue No. 31 of April 12th, 2019. He is born on January 29th, 1972 in Plovdiv. He graduated from Faculty of Mathematics and Informatics of Sofia University Saint Kliment Ohridski in 1996 with Master Degree in Mathematics and Secondary speciality Teacher in Mathematics and Informatics. He defended his PhD thesis "Geometrical Properties of some classes of Banach spaces with unconditional basis" in 2001r. and acquired a PhD educational and scientific degree. His scientific advisor was Acad. Stanimir Troyanski. Since 1999 he is an Assistant Professor at the Chair of Mathematical Analysis of Faculty of Mathematics and Informatics of Plovdiv University Paisii Hilendarski and since 2001 he is an Associate Professor there.

Assoc. Prof. Boyan Zlatanov has submitted for the participation in the contest a full set of paper and electronic materials which complies with the Regulations on the Academic Staff Development of Bulgaria and with the respective regulations of Plovdiv University Paisii Hilendarski.

The scientific interests of Assoc. Prof. Boyan Zlatanov cover a broad variety of topics. In the beginning of his research career he is working in the field of Functional Analysis, more precisely in the Geometry of Banach Spaces. Later his research interests include Fixed Point Theory. It is worth noting that the Functional Analytic methods and the skills and expertise acquired by the candidate in this fundamental area play an important role in the successful research of Assoc. Prof. Boyan Zlatanov in the field of Fixed Point Theory.

Another area of interests of the candidate is related to the introduction of innovative methods in training in mathematics using computer algebraic systems. The monograph of Assoc. Prof. Zlatanov as well as his papers in this field are impressive. The time and work which the candidate put in the

development of creative thinking of the young Bulgarian generation should be highly acknowledged.

Assoc. Prof. Boyan Zlatanov has been a university lecturer for twenty years. Unfortunately, I have not observed his work personally, but it is clearly seen from the materials submitted that he has lectured on a wide range of disciplines. It can be noticed really well reading the list of non-mandatory courses lead by the candidate: fundamental courses in Mathematics (Topology, Functional Analysis), courses in Economics and courses in Education. The candidate is the author of two university textbooks: Mathematical Analysis 1 – Differential calculus of functions of one variable using algebraic computer systems and Mathematical Analysis 2 – Integral calculus of functions of one variable using algebraic computer systems. They are well written and they are rigorous as well as reader friendly. Higher education in Bulgaria needs textbooks on the topic and I have no doubts they are (and going to be) very useful for the students. Although not submitted with the materials for the contest, electronic lecture notes on Metric Spaces are authored by Assoc. Prof. Boyan Zlatanov. There some topics important for every working mathematician are covered in big detail and on an elementary level. I, personally, had a pleasure in reading the great number of interesting problems and exercises which significantly broaden the content and complement the main text.

Assoc. Prof. Boyan Zlatanov submitted for the participation in the contest for Full Professorship 31 scientific articles, two textbooks and a monograph.

I am going to use the numbering of the papers from the list "Scientific papers submitted for the participation in the contest for Full Professorship" in my analysis of the scientific works.

- **Geometry of Banach Spaces**

Assoc. Prof. Boyan Zlatanov submitted for the participation in the contest for Full Professorship 7 scientific articles belonging to this area (published between 2008 and 2019) and he is the author of 16 papers (cited 6 times) in the field. He is the single author of all papers submitted for the contest in this group. Three of them are published in journals with impact factor.

Some sequence spaces of particular type (Köthe, Orlicz, Musielak-Orlicz, Orlicz-Lorentz e.c.t.) are the main object of research in these publications. In the setting of such spaces many general questions in Banach Space Theory can be investigated "in vitro" – using the particular form of the elements of the space. Moreover, the spaces in question can possess a big variety of different properties. The calculations needed are really difficult in most cases and require a lot of techniques and ingenuity. The particular structure of the space allows to find the exact value of some important constants in many instances while in a general Banach space the respective calculations are impossible.

We begin with the results of the candidate which are connected to the

so called normal structure of a Banach space. A Banach space X is said to have normal structure if every bounded convex subset of X which is not a singleton has a non-diametral point (i.e. a point for which the supremum of its distances to the points of the set is strictly less than the diameter of the set). All Banach spaces with normal structure possess weak fixed point property (every nonexpansive map on a convex weak compact to itself has a fixed point). Thus the property of having normal structure relates the different areas of interest of the candidate.

The coefficient $WCS(X)$ (weakly convergent sequence coefficient) of a Banach space X in weighted Orlicz sequences spaces, equipped either with Luxemburg's or Amemiya's norm and a weight sequence $w = \{w_n\}_{n=1}^{\infty}$ belonging to some large class is studied in [1]. It is well known that every reflexive Banach space X with $WCS(X) > 1$ has normal structure. Y. Cui calculated WCS for Orlicz sequence spaces equipped either with Luxemburg's or with Amemiya's norm, provided that M satisfies the Δ_2 condition. The main result of [1] is that a weighted Orlicz sequence space $\ell_M(w)$, equipped either with Luxemburg's or with Amemiya's norm has a weak uniform normal structure if and only if $\ell_M(w) \cong h_M(w)$ for the respective large class of weight sequences $w = \{w_n\}_{n=1}^{\infty}$. A characterization of the weakly null sequences in the considered class of spaces is obtained. An example is constructed, where the Orlicz function M does not have the Δ_2-condition but for a suitable weight sequence $\lim_{n\to\infty} w_n = \infty$ the Orlicz space $\ell_M(w)$ has a weak uniform normal structure. Following the ideas of Changsen and Fenghui, who have defined a generalized modulus of convexity $\delta_X^{(\lambda)}$, a generalized modulus of smoothness $\rho_X^{(\lambda)}$ is introduced in [5] and it is shown that these generalized moduli are connected in a similar way as the classical ones. Some properties of these new moduli are obtained as well as some estimates of these moduli for an arbitrary Banach space. Some inequalities between the $WCS(X)$ coefficient of a Köthe sequence space X and the generalized modulus of convexity $\delta_X^{(\lambda)}$. As a consequence sufficient conditions, connected with the mudulus of convexity, are obtained that ensure that the space has a normal structure.

The packing constant of a Banach space X measures the radius of infinitely many disjoint spheres which can be packed in the unit ball of X. The packing constant and the Kottman constant which is closely related to it are interesting and important parameters when studying the geometrical structure of Banach spaces (isometrical embedding, measures of non-compactness, reflexivity e.c.t.). The packing constant is known for the classical sequence spaces: ℓ_p, Orlicz, Nakano, Musielak-Orlciz, Lorentz, Orlicz-Lorenz, Cesaro. The packing constant of Muscielak-Orlicz sequence space, equipped with Luxemburg's or p-Amemiya's norm is investigated in [7]. A new formula for calculating of the packing constant (and thus of the Kottman constant) in weighted Orlicz sequence spaces with a weight sequence $w = \{w_n\}_{n=1}^{\infty}$ belonging to a large class is presented in [7]. This formula is different from the known one and it is easier to work with.

Borwein and Sims defined the Riesz angle $\alpha(X)$ in a Banach lattice X.

They proved that if the Banach lattice X has the weak orthogonality property and $\alpha(X) < 2$, then it has the weak fixed point property. Reisz angle of ℓ_p and of Orlicz sequences spaces ℓ_M is known. An expression for computing the Riesz angle in weighted Orlicz sequence spaces, equipped either with Luxemburg's or with Amemiya's norm, generated by an Orlicz function M, satisfying the Δ_2-condition and a weighted sequence $w = \{w_n\}_{n=1}^{\infty}$ belonging a large class, is obtained in [6]. The expression for computing the Riesz angle obtained in [6] depends only on the behaviour of the Orlicz function M. It is proved in [7] that for a wide class of Köthe sequence spaces the Kottman constant and the Reisz angle are equal. The exact value of the Reisz angle in the cases when M does not have the Δ_2-condition is found in [7]. Some relations between the Reisz angle and the moduli of convexity and smoothness in Köthe sequence spaces are found again in [7].

The notions of stabilized asymptotic ℓ_p spaces (Milman, Tomczak-Jaegermann) and asymptotic ℓ_p spaces (Maurey, Milman и Tomczak-Jaegermann) are important in the structural theory of Banach spaces and they are related to minimality of a Banach space. Mushielak–Orlicz sequence spaces ℓ_Φ with a dual ℓ_Φ^*, which is stabilized asymptotic ℓ_∞ with respect to the unit vector basis are investigated in [3]. A complete characterization of the bounded relatively weakly compact subsets $K \subset \ell_\Phi$ is obtained. It is proved that ℓ_Φ is saturated with asymptotically isometric copies of ℓ_1 and thus ℓ_Φ fails the fixed point property for closed, bounded convex sets and non-expansive (or contractive) maps on them. It is proven in [4] that if the Muscielak-Orlicz sequence space ℓ_Φ is generated by a Muscielak-Orlicz function, which satisfies the δ_2 condition and its dual space ℓ_Ψ is stabilized asymptotic ℓ_∞ space with respect to the unit vector basis, then ℓ_Φ has the Schur property (this is an earlier result of Boyan Zlatanov) using classical techniques.

The polyhedral Banach spaces (whose unit ball intersected with any finite-dimensional subspace is a polyhedron) are introduced in sixties of the last century and they are studied extensively ever since. Leung proved that an Orlicz sequence space is isomorphically polyhedral if $\lim_{t\to 0} \frac{M(\lambda t)}{M(t)} = \infty$. Dew has shown that if a Muscelak-Orlicz h_Φ sequence space is stabilized asymptotic ℓ_∞ with respect to the unit vector basis, then it is isomorphically polyhedral. Using the ideas of Leung and Dew it is proven in [2] that if the generating Orlicz function M does not have the Δ_2-condition at zero, then the existence of an equivalent analytic norm in the Orlicz-Lorentz sequence space $d_0(w, M)$ is equivalent to $d_0(w, M)$ being isomorphically polyhedral. It is shown that if $\lim_{n\to 0} \frac{M(\lambda t)}{M(t)} = \infty$ for some $\lambda > 1$ then the Orlicz–Lorentz sequence space $d_0(w, M)$ is isomorphic to a polyhedral Banach space and therefore it admits an equivalent analytic norm, it is c_0–saturated and it has a separable dual.

• Fixed Point Theory and Best Proximity Points

Assoc. Prof. Boyan Zlatanov submitted for the participation in the contest for Full Professorship 15 scientific articles belonging to this group and

he is the author of 22 papers (cited 48 times) in the field. He is the single author of three papers submitted for the contest in this group and the rest of them are joint works with Bulgarian or foreign mathematicians. Nine of these articles are published in journals with impact factor.

It is worth noting once more that that the first and the second group of publications of the candidate are closely related. It is good for the younger colleagues to understand that the knowledge of infinite-dimensional spaces gives a big advantage in a large part of applied research areas. No doubt that Assoc. Prof. Boyan Zlatanov has this advantage. On the other hand specific applied problems stimulate and broaden fundamental research.

Fixed point theory is an important part of contemporary mathematics. The main aim is to solve equations of the kind $Tx = x$ for mappings, defined on subsets of metric spaces or normed spaces. The main classical results are Banach contraction principle and Brouwer fixed point theorem. The proof of the second mentioned theorem is non-constructive (although there exist numerical methods based on its ideas) and, moreover, it is restricted strictly to the finite-dimensional case. The proof of Banach contraction principle is very simple and, besides existence, it provides uniqueness of the fixed point, constructive method for finding it and an error estimate. That is why so much effort is put in attempts to generalize Banach contraction principle not loosing much of these advantages. A generalization of the Banach contraction principle is the notion of cyclical maps introduced by Kirk, Srinivasan and Veeramani. Because a non-self mapping does not necessarily have a fixed point, one often attempts to find an element x, which is in some sense closest to Tx. Best proximity points, introduced by Eldrad and Veermani, are relevant in this perspective. The idea to consider a cyclic map between two closed convex disjoint sets originates in the results of Eldred and Veermani. Later Karpagam and Agrawal generalized this idea for cyclic maps between p sets but the condition imposed on the map seems to be too restrictive, as it is satisfied only when the distances between the consecutive sets are equal. The notion of p-summing maps is introduced in [9]. This new type of a contractive condition ensures the existence and uniqueness of fixed points and best proximity points in uniformly convex Banach spaces and the distances between the consecutive sets may be different. Fixed point theorems for Kannan and Zamfirescu operators in the presence of a cyclical contractive condition are obtained in [8]. A method for the approximation of the fixed points is also provided, for which both "a priori" and "a posteriori" error estimates are given. These results generalize, unify and extend several important fixed points theorems in the literature. The idea to consider summing type conditions is further developed for reflexive spaces in [19]. New sufficient conditions are found that ensure the existence and uniqueness of best proximity points in uniformly convex Banach spaces for iterated p-summing contractions. Sufficient conditions for the existence and uniqueness of best proximity points in reflexive Banach spaces for iterated p-summing contractions are obtained which generalize some known results in the literature.

Variational techniques is widely used in contemporary mathematics, in particular in fixed point theory. In [13] an appropriate generalization of Eke-

land's Variational Principle for cyclic maps is found which allows for using variational techniques when investigating best proximity points. Interesting applications of this version of the variational principle are presented in the paper.

Another research field is obtaining error estimates for the sequence of successive iterations. Assoc. Prof. Boyan Zlatanov proved "a priori error estimates" and "a posteriori error estimates" for finding best proximity points for different types of cyclic maps in [14] on a uniformly convex Banach space with modulus of convexity of power type. This result fills a gap in the existing literature. These ideas are further developed in [22] where sufficient condition for the existence and uniqueness of best proximity points for a cyclic Reich contraction map (defined on a uniformly convex Banach space with modulus of convexity of power type) is found and "a priori" and "a posteriori" error estimates of the best proximity point are obtained.

Further developing Karpagam-Agrawal work on orbital contractions for cyclic Meir-Keeler maps, it is introduced in [11] the notion of p-summing orbital Meir-Keeler contractions. By proposing two p-summing conditions the author gets necessary and sufficient conditions for the existence and uniqueness of best proximity points. Using the technique of L-functions, introduced by Lim and further developed by Suzuki, sufficient conditions for the existence and uniqueness of best proximity points and fixed points for p-cyclic, orbital Meir-Keeler maps are obtained in [15].

A direction for the generalization of some fixed point results is considering operators whose domain has not a Banach space (or, more generally, complete metric space) structure. Khamsi, Kozlowski and Reich were the first to present such a generalization in modular function spaces (function spaces where the growth of the functions is controlled not by a norm but by a convex even functional satisfying some conditions). In [12] appropriate generalization of the notion of best proximity points in the context of modular function spaces is given. Sufficient conditions for the existence and uniqueness of best proximity points for cyclic maps in modular function spaces are found. It happens that some additional research on the geometry of the modular function spaces, which may have very strange structure, is needed. Developing these ideas, in [18] the notion of best proximity points and the notion of fixed points for cyclic contraction Kannan maps in modular function spaces is generalized. Sufficient conditions for the existence and uniqueness of best proximity points and fixed points for cyclic Kannan maps in modular function spaces are found. As corollaries sufficient conditions for the existence and uniqueness of best proximity points and fixed points for cyclic maps in Orlicz spaces, endowed with an Orlicz function modular are obtained.

Continuing the idea to generalize best proximity results by changing the underlying space is to consider maps defined on a partially ordered complete metric space (the distance of a point to itself may be positive). Meir-Keeler contractions are generalized for partially ordered metric spaces in [20]. Sufficient conditions for the existence and uniqueness of fixed points and the best proximity points for these maps in complete partial metric spaces are ob-

tained. If the underlying space is a b–metric space (in the triangle inequality the right-hand side is multiplied by the so called b-metric constant), in [16] sufficient conditions for the existence and uniqueness of fixed points of Chatterjea's maps in b-metric space are found. These conditions do not involve the b-metric constant, but it is required that the set of the orbits of the successive iterations is bounded, a condition widely used in the fixed point theory in modular function spaces. The authors establish a priori error estimate for the sequence of successive iterations (better than the well known one for a wide class of Chatterjea's maps in metric spaces). The same is done in [17] for a class of Reich maps in b-metric space. Thus the results in [16] turn to be corollaries of the main result of [17].

A p-cyclic orbital Geraghty type of contraction is introduced in [21] and sufficien conditions for existing and uniqueness of a best proximity point is obtained in a uniformly convex Banach space setting. A result about fixed points of classical maps is obtained in [10]. The results of Sehgal and Guseman with a contractive iterate condition at each point are generalized for mappings on a complete metric space.

• **Usage of dynamic geometry software in education**

Assoc. Prof. Boyan Zlatanov submitted for the participation in the contest for Full Professorship 8 scientific articles belonging to this group and he is the author of 13 papers (cited 46 times) in the field. The monograph of the candidate belongs to this area as well. Unfortunately, I am not competent enough to assess the achievements in this research field. Nevertheless, I am impressed by the scope of the pedagogical interests of the candidate and by the detailed and in-debth presentation of the material.

Assoc. Prof. Boyan Zlatanov submitted for the participation in the contest for Full Professorship 100 citations of his works. Fifty-nine of them are published in reputable journals (refereed in Web of Science and/or SCOPUS). These numbers are much greater than the national minimum requirements as well as the Faculty of Mathematics and Informatics specific requirements, approved in relation with the Plovdiv University Regulations on the Implementation of the Law on the Scientific Staff Development in the Republic of Bulgaria.

I have no doubts that Assoc. Prof. Boyan Zlatanov has contributed at least an equal share to all joint publications.

No plagiarism was found in the publications submitted for assessment.

The candidate Assoc. Prof. Boyan Zlatanov has a PhD degree, he is listed in the list of habilitated persons as an Associate Professor in NACID. He has presented a monograph and publications with total of 319 points for group B, publications with a total 606 points for group Г, citations with total

472 for group Д and 200 points for group E. All of the points exceed the minimum requirements according to the Rules for applying of the Law on the Development of the Academic Staff in the Republic of Bulgaria. The candidate participates in the contest with total of 31 scientific publications (30 in journals and 1 in proceedings of a conference), where 12 are published in journals with an IF, thus he satisfies the additional requirements of the Faculty of Mathematics and Informatics at University of Plovdiv "Paisii Hilendarski" (20 publications, 12 of them to be in journals and 8 of them to be published in journals with an IF). The candidate presents 100 citations thus he satisfies the additional requirements of the Faculty of Mathematics and Informatics at University of Plovdiv "Paisii Hilendarski", where 20 citations are needed. The candidate has a PhD student, who successfully obtained his degree in the Faculty of Mathematics and Informatics within the University of Plovdiv "Paisii Hilendarski". The candidate presents 2 textbooks, thus he satisfies the additional requirements of the Faculty of Mathematics and Informatics at University of Plovdiv, where 1 textbook or 1 lecture notes are needed. It is clearly seen that the achievements of Assoc. Prof. Boyan Zlatanov always comply and in most cases significantly exceed all requirements for occupying the position of full professor at Faculty of Mathematics and Informatics of University of Plovdiv "Paisii Hilendarski".

The scientific achievements of Assoc. Prof. Boyan Zlatanov leave no doubt in the positive assessment of his research and pedagogical activity.

I recommend to the Scientific Jury to prepare a report-proposal to the Honorable Scientific Faculty Council of the Faculty Mathematics and Informatics for the election of Assoc. Prof. Boyan Georgiev Zlatanov, PhD for the academic position Full Professor in Plovdiv University "Paisii Hilendarski" on Research area: 4. Natural Sciences, Mathematics and Informatics, Professional field 4.5 Mathematics (Mathematical Analysis).

10.09.2019

(Prof. N.Ribarska, DSci)