Review

by Professor Sci. D. Geno Petkov Nikolov, Faculty of Mathematics and Informatics of Sofia University St. Kliment Ohridski

of the materials submitted for participation in the contest for occupation of the academic position *Professor* in the Faculty of Mathematics and Informatics of Plovdiv University Paisii Hilendarski in Area of Higher Education 4. Natural Sciences, Mathematics and Informatics, Professional Field 4.5 Mathematics (Mathematical Analysis), announced in *State Gazette* No 31/12.04.2019

1 Subject of assessment

In the contest for the academic position *Professor* announced in *State Gazette* No 31/12.04.2019 and on the web site of Plovdiv University Paisii Hilendarski (in short, henceforth abbreviated as PU) for the needs of Faculty of Mathematics and Informatics participates only one candidate: Associate Professor Dr. Boyan Georgiev Zlatanov.

By Order No P33-3379/12.07.2019 of the Chancellor of PU I was appointed as a member of the Scientific Jury of the aforementioned contest. By a decision of this Scientific Jury from 18.07.2019 I was assigned to write a review of the contest.

As a member of the Scientific Jury I have received from Associate Professor Dr. Boyan Georgiev Zlatanov in the paper and electronic format the following documents, which he submitted along with Application Form to the Chancellor of PU on 28.06.2019:

2. Curriculum Vitae (in Bulgarian and English);
3. Photocopy of Diploma for Master's degree A-95 No 130597 issued by Sofia University (SU) along with an Official Notice from the Faculty of Mathematics and Informatics of SU;
4. Photocopy of PhD Diploma No 27519/27.08.2001, issued by the Higher Attestation Committee;
5. Photocopy of Diploma No 24922/23.04.2008 for Associate Professor in Mathematical Analysis;
6. Screen shot from the NACID web site confirming applicant's compliance with the minimal national requirements;
7. Work Experience Certificate У-1829/05.04.2019;
8. Teaching Experience Documents (Teaching Courses after occupying Assoc. Professor position and a list of Textbooks published);
9. Reference from the Scientific and Project Activity Office of PU about applicant's participation in Scientific and Educational Projects;
10. Lists of applicant's scientific publications, including: 10.1 list of all papers; 10.2 list of the papers deposed for acquiring PhD Degree; 10.3 list of the papers deposed for acquiring Associate Professor position; 10.4 list of the papers deposed for participation in the Contest; 10.5 copies of the latter papers; 10.6 list of the citations of applicant's work deposed for participation in the Contest; 10.7 list of papers which have not been deposed by the applicant in preceding promotions and in the present Contest; 10.8 Evidences for citations of applicant's work deposed for the Contest (in electronic format only);

11. References confirming compliance with: 11.1 the minimal national requirements for Scientific Field 4.5; 11.2 the specific requirements of the Faculty of Mathematics of PU; 11.3 the additional requirements of the Faculty of Mathematics of PU according to art. 76.(4) of the Rules for the application of the Development of Academic Staff in the Republic of Bulgaria Act (DASRBA) of PU;

12. declaration of originality and authenticity of the deposed documents;

13. Abstracts of the papers and self-assessment of the contributions according to art. 76.(4) of the Rules for the application of DASRBA of PU (in Bulgarian and in English).

The deposed set of documents and materials contributes for the objective and complete assessment of the candidate according to the Development of Academic Staff in the Republic of Bulgaria Act (DASRBA) and the Rules for its application as well as the regulations of Plovdiv University Paisii Hilendarski and of Faculty of Mathematics and Informatics.

2 Biographical data, teaching and project activity of the applicant

The applicant Boyan Georgiev Zlatanov graduated in 1995 from Sofia University St. Kliment Ohridski in Mathematics, specialty Mathematical Analysis. In 2001 Zlatanov acquired PhD Degree with a PhD Thesis entitled *Geometric Properties of Certain Banach Spaces with Unconditional Basis*. From 1999 by now his teaching activity has been permanently based in PU. Dr. Zlatanov worked as Assistant Professor in PU until 2008, when he was promoted to Associate Professor. Thus, the applicant meets the additional requirements for Work Experience postulated in art. 76 (3, 4) of the Rules for application of DASRBA of PU. His teaching activity features diversity of subjects: he has prepared 18 courses, including the compulsory courses of *Mathematical Analysis 1, 2, 3 and 4*, as well as the facultative courses *Functional Analysis*, *Applied Functional Analysis*, *Geometry of Fractals*, *Metric Spaces*, *Convergence and Compactness in Metric Spaces*, *Exchange and OTC Trading*, *Introduction to Mathematics of Money*, *Value Theory in Market Economy*, *Computer Algebraic Systems*, *GeoGebra - Potentialities and Challenges*, etc. From 2015 by now Dr. Zlatanov is Deputy Dean on the Research and Applied Activity in the Branch of PU in Smolyan. Dr. Zlatanov has been adviser of the Diploma Works of 8 Master Degree students, and of the Ph.D. student A. Ilchev, who defended his Ph.D. Thesis in the Faculty of Mathematics and Informatics (FMI) of PU. With the latter Zlatanov meets one of the additional requirements for applicants for position Professor in FMI.

The applicant participated in two national research projects and in one national educational project, as well as in four research projects of PU. He has been member of Programm and/or Organizing Committees of 5 scientific conferences, and participated with talks in 12 conferences and seminars. Boyan Zlatanov is a member of the editorial boards of two journals, one of them published in Bulgaria and one abroad. He is reviewer in Math Review as well as in 15 specialized journals.

3 General description of applicant's publications

The list of all publications of Assoc. Professor Boyan Zlatanov consists of 53 papers, two textbooks on differential and on integral calculus with application of computer algebraic systems, one monograph and electronic materials (lecture notes). 18 of the papers are in journals with impact factor. In the Contest for Professor the applicant participates with 31 papers, the two textbooks and the monograph. All the deposed publications have been published after his habilitation. 30 of the papers are in journals and one in proceedings of a conference. 12 of the papers are published in recognized

international journals with impact factor, amongst them are *Journal of Mathematical Analysis and Applications*, *Nonlinear Analysis: Modelling and Control* (2), *Fixed Point Theory and Applications* (2), *Acta Mathematica Sinica*, *Mathematica Slovaca*, *Carpathian Journal of Mathematics*, etc. Hence, the applicant fulfills the additional requirements of FMI (PU) for publications (minimum 20 papers, at least 12 of them in journals, at least 8 of them with impact factor, at least one published textbook or lecture notes).

The applicant has presented evidence for 100 citations of his work, among them 59 citations in journals indexed by Web of Science and/or Scopus (here, the specific requirements of FMI (PU) is for minimum 20 citations).

The deposed by Associate Professor Dr. Boyan Zlatanov publications may be classified into three groups:

- Some geometric properties of Banach spaces with unconditional basis. These are papers [1]–[7] (here and henceforth we follow the enumeration from the list 10.4);
- Fixed point theory and best proximity points for cyclic mappings: papers [8]–[22];
- Application of algebraic computer systems and dynamic geometric software and Maple in the education in mathematics: papers [24]-[31]. The applicant's monograph *Developing Creative Thinking in Geometry Classes*, published by LAP LAMBERT Academic Publishing, also falls in this group.

Aside of these groups is [23], where methods from mathematical analysis are applied to investigate a problem from electrostatics.

4 Analysis of the scientific contributions of the applicant

4.1 Some geometric properties of Banach spaces with unconditional basis

The papers [1, 2, 3, 4, 5, 6, 7] are in the field of geometry of Banach spaces. Various properties and characteristics of classes of Köthe sequence spaces and Musielak–Orlicz sequence spaces are studied in these papers. Amongst these characteristics are the Kottman constant and the packing constant, the Reisz angle, the weakly convergent sequences coefficient, the moduli of convexity and of smoothness and some of their generalizations, the Schur property, the normal structure, etc. For various classes of Köthe sequence spaces are obtained results connected with the property the space to be stabilized asymptotic ℓ_∞.

The subject of study in [1] is $WCS(X)$, the weakly convergent sequences coefficient of a Banach space X, introduced by Bynum. The value of WCS is known for ℓ_p, c_0 and for Hilbert spaces. In [1] WCS is studied in weighted Orlicz sequence spaces endowed either with the Luxemburg or the Amemiys norm, and weight sequences $w = \{w_n\}_{n=1}^{\infty}$ from classes Λ or Λ_∞. A weight sequence is from the class Λ_∞, if it is non-decreasing and converges to infinity. For a wide class of weight sequences $w = \{w_n\}_{n=1}^{\infty}$, including Λ и Λ_∞, Zlatanov proves a necessary and sufficient condition for the weighted Orlicz sequence spaces $\ell_M(w)$, equipped with either with the Luxemburg or the Amemiys norm, to have weak uniform normal structure.

In [5] Zlatanov introduces a generalized modulus of smoothness $\rho_X^{(\lambda)}$ ($\lambda \in (0,1)$), influenced by the ideas of Changsen and Fenghui, who introduced a generalized modulus of convexity $\delta_X^{(\lambda)}$. He showed that the generalized moduli of convexity and smoothness are connected in an analogous way as the classical ones, which generalizes the Lindenstrauss result, and proves estimates for these moduli for arbitrary Banach space and for the particular case $X = \ell_p$. The main result here is a sufficient condition, expressed in terms of the modulus of convexity, for a wide class of Köthe sequence spaces with restricted complete and shrinking basis to have normal structure. This generalizes a result of Gao and Lau.

The notion of asymptotic ℓ_p spaces is due to Milman and Tomczak–Jaegermann, and the collection of such spaces is nowadays known as stabilized asymptotic ℓ_p spaces. In [3] Zlatanov studies the Musielak–Orlicz sequence spaces ℓ_Φ with a dual ℓ_Φ^*, which is stabilized asymptotic ℓ_∞ with respect to the unit vector basis. A complete characterization is given of the bounded relatively weakly compact subsets $K \subset \ell_\Phi$. He proved that ℓ_Φ is saturated with asymptotically isometric copies of ℓ_1 and therefore does not possess the fixed point property for closed bounded and convex sets and non-expansive (or non-contractive) maps on them. The results are illustrated with examples.

The fact that every weakly null sequence ℓ_1 converges to zero also in norm is known as the Schur property. In a 2007 paper (not included in the applicant's list for this contest) Zlatanov showed that if the Musielak–Orlicz sequences space ℓ_Φ is generated by a Musielak–Orlicz function which satisfies the δ_2–condition and its dual space ℓ_Ψ is stabilized asymptotic ℓ_∞ with respect to the unit vector basis, then ℓ_Φ possesses the Schur property. In [4] the applicant presents another proof of this result using classical methods of functional analysis with a flavor of the Banach times.

By exploiting some ideas of Leung and Dew, Zlatanov proves in [2] that if the generating Orlicz function M does not satisfy the Δ_2–property at zero, then the existence of an equivalent analytic norm in the Orlicz–Lorentz sequence space $d_0(w, M)$ is equivalent to the condition the space $d_0(w, M)$ to be isomorphically polyhedral. Furthermore, he proved that if $\lim_{n\to 0} \frac{M(\lambda t)}{M(t)} = \infty$ for some $\lambda > 1$, then the Orlicz–Lorentz sequence space $d_0(w, M)$ is isomorphic to a polyhedral Banach space and therefore it admits an equivalent analytic norm; it is c_0–saturated and has a separable dual. Through either the generating Orlicz function or the existence of isomorphic copies of ℓ_p are characterized all the c_0–saturated Orlicz–Lorentz sequence spaces. A class of Orlicz–Lorentz sequence spaces is constructed, which is isomorphically polyhedral, admits an equivalent analytic norm and is c_0-saturated.

In [6] Zlatanov investigates the Riesz angle $\alpha(X)$ in Banach lattices, introduced by J. Borwein and B. Sims. He finds a new formula for the evaluation of the Riesz angle in weighted Orlicz sequence spaces endowed with either the Luxemberg norm or with the Amemyia norm, generated by an Orlicz function, which satisfies the Δ_2–property, and a weighted sequence $w = \{w_n\}_{n=1}^\infty$ from the class Λ. The Zlatanov formula for the evaluation of the Riesz angle depends only on the behavior of the Orlicz generating function. For some classical (but fairly non-trivial) Orlicz functions he shows when the Riesz angle can be exactly evaluated. In the second case (Amemyia norm) he proves two-sided estimates for the Riesz angle.

The paper [7] studies the packing constant and the Kottman constant, introduced by Kottman in the seventies of the last century. In contrast to the case of finite-dimensional spaces, where the unit ball may contain only finite number of non-overlapping balls with equal radii, for a Banach space X of infinite dimension there exists a constant Γ_X (packing constant) with the property that the unit ball $B(X)$ may contain infinitely many non-overlapping balls with the same radius, provided this radius is less than Γ_X. It is known that $\Gamma_X \in [1/3, 1/2]$, and Γ_X is found for certain sequence spaces. The Kottman constant $K(X)$ for a infinite-dimensional Banach space X is related to the packing constant by the equality $\Gamma_X = \frac{K(X)}{K(X)+2}$. In [7] Zlatanov found a new formula for the evaluation of the packing constant and thereby of the Kottman constant for weighted Orlicz sequence spaces with weight sequences $w = \{w_n\}_{n=1}^\infty$, belonging to the class Λ and equipped with either the Luxemberg norm or the p–Amemyia norm. The formula found by Zlatanov is easier to work with and enables through the packing constant to evaluate the Kottman constant for weighted Orlicz sequence spaces with weight sequences from the class Λ. Zlatanov shows that for a wide class of Köthe sequence spaces the Kottman constant and the Riesz angle are equal. This class encompasses the spaces that are order continuous with the Fatou property and the spaces where the unit vector basis is unconditional and boundedly complete. This allows the evaluation of the Riesz angle via the packing constant whenever the latter is known.

4.2 Fixed points theory and best proximity points for cyclic maps

As was already mentioned, this group consists of papers [8]–[22]. It is worthy mentioning that, on the one hand-side, these papers are based on certain results from the geometry of Banach spaces and, on the other hand-side, some of the results in these papers contribute and enrich the theory in the geometry of Banach spaces.

Fixed points are important tool for solving equation $Tx = x$ for maps defined in metric or normed spaces. The notion of contracting cyclic operator, introduced by Kirk, Srinivasan and Veermani, is an appropriate extension of the Banach contracting principle. A cyclical map may not have a fixed point, that is why it is the reasonable to search for an element x, which in a certain sense the closest to Tx. Eldred and Veermani are the first to consider cyclic maps between two closed convex and non-intersecting sets, they defined the so-called best proximity points in a set, which naturally generalizes the notion of fixed points. The idea for cyclic maps between $p \geq 2$ sets was introduced later by Karpagam, and Agarwal. The condition they imposed on the cyclic maps however seems too restrictive as it can be satisfied only when the distances between the consecutive sets are equal. In [8] Zlatanov and Petric found sufficient conditions for the existence and uniqueness of fixed points of the cyclic maps of Kannan and Zamfirescu, and proved a priori and a posteriori error estimates. In [9] the same authors define p–cyclic summing maps, imposing a new type contractive condition, which provides the existence and uniqueness of fixed points or best proximity points in uniformly convex Banach spaces also in cases when the distances between the consecutive sets are different. The results of Eldred, Veermani and Karpagam are obtained as special cases of the results in [9]. Appropriate examples of p–cyclic summing maps are given, where the distances between the consecutive sets are different.

The idea of p–cyclic summing maps is further developed for reflexive spaces in [19]. Sufficient condition are proved for the existence and uniqueness of the best proximity points of p–cyclic summing maps in uniformly convex Banach spaces, and also in reflexive Banach spaces, which extend related results of other authors (Thagafi and Shahzad, Karpagam and Agarwal, etc.)

In [11] Zlatanov introduces the notion p–cyclic summing orbital Meir–Keeler contractive maps. By imposing two summing conditions, he obtains some sufficient conditions for the existence and the uniqueness of the best proximity points, thus extends earlier results of Karpagam and Agarwal. Next, in a joint paper with Karpagam [15] Zlatanov applies the technique of L–functions to obtain sufficient conditions for the existence and the uniqueness of the best proximity points for p–cyclic orbital Meir–Keeler contractive maps.

Finding error estimates for the best proximity points for sequences of successive iteration is a worth to study problem. The first result in this direction was obtained by the applicant in [14], where *a priori* and *a posteriori* error estimates for the cyclic maps introduced by Eldred and Veermani are proved, in terms of the modulus of convexity. In [22] Zlatanov and Ilchev obtain sufficient conditions for the existence and uniqueness of best proximity points for cyclic contractive maps of Reich, thus extending some known results about cyclic maps of Kannan and Chatterjea. *A priori* and *a posteriori* error estimates of the best proximity points are proved for sequences of successive iterations, when the space is uniformly convex with a modulus of convexity of power type. As a consequence, they estimate the error for the cyclic contractive maps of Kannan and Chatterjea.

A step towards the generalization of the fixed points is to change the underlying space. In [12] and [18] the concept of cyclic maps, fixed points and best proximity points is studied in modular functional spaces. The requirement for uniform convexity here is replaced by the conditions $UC1$ and UC. The results in these papers are of independent interest, as they enrich the knowledge about the geometry of modular functional spaces. Some sufficient conditions for the existence and uniqueness of fixed points and of best proximity points for the case of partially ordered complete metric space are obtained in [20]. In [21] these problems are considered for p–cyclic orbital Geraghty type contractive maps in uniformly convex Banach spaces. It is proved there that the best proximity point is also the unique periodic poind for the maps under consideration.

Ekeland (1976) introduced a variational principle which has found numerous applications in various fields of mathematics, including the fixed point theory. In the (joint with M. Ivanov and N. Zlateva) paper [13] the Ekeland variational principle is extended for cyclic maps and applied to the proof of the existence and uniqueness of best proximity points for certain classes of cyclic maps.

In [10] the authors obtain a result of the existence and uniqueness of fixed points in complete metric spaces with contractive iterative property, which implies the results obtained earlier by Seghal and Guseman. An example illustrating the result is constructed.

The papers [16, 17] study the existence and the uniqueness of fixed points in b-metric spaces. The difference of these spaces with the classical metric spaces is that the triangle inequality for the distance function is weakened (the right-hand side of this inequality is multiplied by a constant $s \geq 1$). The results in [17] extend that in [16], providing some sufficient conditions for the existence and uniqueness of fixed points for classes of Reich maps, which do not depend on the constant s. The error estimates in [17] improve upon the known ones in metric spaces.

Aside of the topics of the above two groups of papers is the paper [23]. There the authors analyze a formula, describing the electrostatic interaction between two charged conducting spheres with equal radii and charges, obtained by Kolikov, Ivanov, Krustev, Epitropov and Bozhkov. It is proved for the correction coefficient, introduced by these authors to complement the Colomb law, that it is less than 1 when the ratio of the radius and the distance between the spheres is less than $\frac{2}{5}$. It is also shown that this correction coefficient can be evaluated with an arbitrary prescribed.

4.3 Usage of computer algebraic systems and dynamic geometric software and Maple in education in mathematics

The third group of papers [24]-[31] and the monograph [32] reflect the deep interest of Associate Professor Dr. Boyan Zlatanov in the implementation of *Algebraic Computer Systems* (ACS), and in particular *Dynamic Geometry Software* (DGS) in the education in Mathematics. The benefits from the usage of such systems are beyond any doubt and include: 1) optimization of the teaching process; 2) integration of the secondary school and university training and 3) development creative thinking in the classes of mathematics, and especially in geometry classes.

I am not going to analyze in detail this group of publications of the applicant, and it is not because I underestimate their significance and quality. On the contrary, these publications best introduce Dr. Zlatanov as a dedicated lecturer who fruitfully uses the advantages of the modern IT age in both the educational process and the research. I am tempted to say that with the professionally developed DGS *Sam* and his skills in using DGS GeoGebra, with the publications on their implementation and application for solving various interesting geometric problems, also described in his monograph, deep learning of the ACS *Maple* (demonstrated in the two textbooks on differential and integral calculus), Boyan Zlatanov could be a successful candidate for Professor in Professional Field *4.6 Informatics and Computer Science*, why not also in *3.1 Pedagogy of Education in ...*

The reason for abstaining from detailed analysis of the papers in this group is because I am profoundly convinced that the scientific publications of Associate Professor Dr. Boyan Zlatanov in the first two groups are well enough both as quantity and quality for occupying position *Professor* in Professional Field *4.5 Mathematics* not only in Plovdiv University Paisii Hilendarski, but also in any other University in Bulgaria.

5 Conclusion

The documents submitted for the Contest by Associate Professor Dr. Boyan Zlatanov show that the Applicant pedagogical experience and research competencies comply with the Contest specificity. In compliance with the prescriptions of the Development of Academic Staff in the Republic of

Bulgaria Act (DASRBA) and the Rules for its application, I declare my conviction that the results claimed by Dr. Zlatanov in his publications are achievements of the author, and did not find any plagiarism.

On the basis of the analysis made above I conclude that the applicant meets in full the requirements of the Development of Academic Staff in the Republic of Bulgaria Act (DASRBA) and the Rules for its implementation, the relevant Rules of Plovdiv University Paisii Hilendarski postulated in art. 76 as well as the specific requirements of the Faculty of Mathematics and Informatics, for occupying position *Professor* in *Professional Field 4.5 Mathematics*.

All this gives me the reason to give my positive assessment to the applicant and recommend to the respected Scientific Jury to prepare a report-proposal to the Scientific Council of the Faculty of Mathematics and Informatics for the election of Associate Professor Dr. BOYAN GEORGIEV ZLATANOV for the academic position PROFESSOR in Plovdiv University Paisii Hilendarski in Professional Field *4.5 Mathematics*, Specialty *Mathematical Analysis*.

Date: 9.09.2019 Reviewer Signature:

(G. Nikolov)